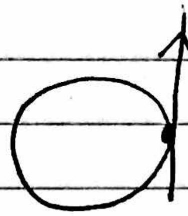
L13 B exp ctd 18

Examples

Case $G = S^1$



Think of $X = i$ as basis
$t \to e^{it} = \cos t + i \sin t$.

$G = (\mathbb{R}, +)$; $\mathfrak{g}$.

$\exp(1t) = t$, $1 \in \mathfrak{g} = \mathbb{R}$ as 'X' as basis.

$SO(3)$:

$$so(3) = \begin{pmatrix} 0 & \omega_3 & -\omega_1 \\ -\omega_3 & 0 & \omega_2 \\ \omega_1 & -\omega_2 & 0 \end{pmatrix} = A ; \; A^T = -A.$$

$$\simeq (\mathbb{R}^3, \times) \; ; \; [v, w] = v \times w.$$

$$A_{\vec{\omega}}(x) = \vec{\omega} \times \vec{x} \; , \; A_{\vec{\omega}} : \mathbb{R}^3 \to \mathbb{R}^3$$

verify:

$$\langle A_\omega x, v \rangle = \langle x, A_\omega v \rangle$$

$$(\vec{\omega} \times \vec{x}) \cdot \vec{v} = -\vec{x} \cdot (\vec{\omega} \times \vec{v})$$

[triple prod.

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$e^{t\vec{\omega}} = \begin{cases} \text{rotation about axis } \vec{\omega} \\ \text{by } t|\vec{\omega}| \text{ radians} \end{cases}$

so

$R = e^{t\omega}$ s.t. : $R\vec{\omega} = \vec{\omega}$

& $R|_{\vec{\omega}^\perp}$ = rotation by $t|\vec{\omega}|$.

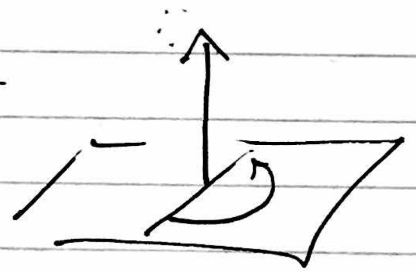
<u>Cor</u>: $SO(3) \simeq \mathbb{R}P^3$.

<u>Pf</u>. $\exp: B^3(\pi) \xrightarrow{\text{onto}} SO(3)$

every rotation is a rotation about <u>some</u> ~~angle~~ axis by <u>some</u> angle.

$\exp(0) = Id$)

But



rotating by π about unit $\vec{u}$ axis same as rotating by π about $-u$ axis

coord: $(x, y, z) \mapsto (-x, -y, z)$

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so $\exp(\pi\vec{u}) = \exp(-\pi\vec{u})$

$|\vec{u}| = 1.$

$\Rightarrow$: Form solid ball B^3
& identify antipodal pts
get: $SO(3)$!

Another way use

$$SU(2) = Sp(1) = \text{unit quaternions}$$
$$\big\downarrow 2{:}1$$
$$SO(3)$$

&