L 12 C

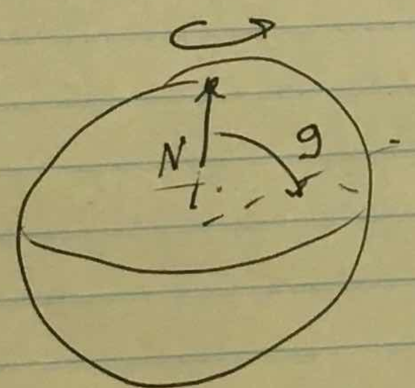
Homogeneous spaces & Lie groups.

Egs

1. $X = S^2$, $G = SO(3)$

$$\begin{array}{ccccc} & G & & g & \\ S^1 & \downarrow & & \downarrow & \\ & X & & g \cdot N & \end{array}$$

fiber: $S^1 = SO(2)$



2. $X = \mathbb{C}P^1$ $(= S^2)$

$G = GL(2, \mathbb{C})$ or $SL(2, \mathbb{C})$
or $\mathbb{P}GL(2, \mathbb{C})$

$[(z_1, z_2)] = [z] \mapsto [gz]$

projective linear transf

In affine coord

$$z \longmapsto \frac{az+b}{cz+d}\ ; \quad X \simeq G \Big/ \left\{\begin{pmatrix} a & c \\ 0 & b \end{pmatrix}\right\}$$

3. $X = \mathbb{R}^2$, $G = SE(2)$
$= \mathbb{R}^2 \rtimes S^1$.
translations & rotations.

$$X = G/S^1$$

4. $X = \mathbb{H}^2 =$ Poincare upper $\frac{1}{2}$ plane.

$G = SL(2, \mathbb{R})$.
$\simeq$ unit tgt bundle of $\mathbb{H}^2$

1, 3, 4 : the group acts freely & transitively on unit vectors, so
$$G \simeq T_1 X.$$

Basic eqn of const. curvature

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$G = U(n)$

$X =$ Grassmann.

or

$X =$ Flag manifold.