vector bundles.

subbundles

$$\begin{array}{ccc} T^*Q^n & & TQ^n \\ \cup & & \cup \\ \mathcal{I} = D^\perp & \xleftrightarrow{\text{pairing.}} & D \end{array}$$

locally: $\theta^1, \theta^2, \ldots, \theta^c$ $\qquad X_1, X_2, \ldots, X_k$

$$k + c = n.$$

$$\theta^i(X_j) = 0.$$

dual version, Fröb, $\quad$ Th'm $\quad$ Fröb.

integrable: $d\mathcal{I} \subset \mathcal{I}. \iff [D, D] \subset D.$

$\mathcal{I}$ = ideal in $\Omega(Q)$ gener by θ

---

eg: a single 1-form

$$d\theta \in \mathcal{I} \iff d\theta = f\,\theta.$$
$$\iff \theta \wedge d\theta = 0.$$

Pf Thm

uses only:

$$d\theta(v,w) = v\theta(w) - w\theta(v) - \theta([v,w])$$

$\Rightarrow$ $\mathcal{D}$, given w/ $[\mathcal{D},\mathcal{D}] \subset \mathcal{D}$.

local frame X_a of v-flds

so $X_a(q) \in \mathcal{D}_q$.

& $[X_a, X_b]_q = \sum c_{ab}^d(q) X_d(q)$. $(*)$

θ^μ annihilating forms

$$\theta^\mu(X_a) = 0, \quad \mu = 1, \dots c$$
$$a = 1, \dots K.$$

$$d\theta^\mu(X_a, X_b) = X_a \underbrace{\cancel{\theta^\mu(X_b)}}_{0} - X_b \underbrace{\cancel{\theta^\mu(X_a)}}_{0}$$
$$- \theta^\mu([X_a, X_b])$$

Now ~~since~~ by $(*)$

$$d\theta^\mu(X_a, X_b) = -\theta^\mu\left(\sum c_{ab}^d X_d\right)$$
$$= -\sum c_{ab}^d \theta^\mu(X_d)$$
$$= 0.$$

Have proved: if $\mathcal{D} \subset [\mathcal{D}, \mathcal{D}]$ then

$\theta \in \mathcal{D}^\perp \Rightarrow d\theta$ kills $\Lambda^2 \mathcal{D}$.

lin. multiples of $X_a \wedge X_b$

$X_a, X_b \in \mathcal{D}$.

linear algebra at a point: Now.

$S \subset V$. $\quad S^\perp \subset V^*$.

$\omega \in \Lambda^2 V^*$.

$\omega(s_1, s_2) = 0 \quad \forall\ s_1, s_2 \in S$

$\Leftrightarrow \omega = \sum \theta^i \wedge \alpha_i$

θ^i basis for $S^\perp$.

$\Leftrightarrow \omega \equiv 0 \mod S^\perp$

$\Leftrightarrow \omega \in \langle S^\perp \rangle$

[us: $d\theta^a = \sum c^a_{be}\, \theta^b \wedge \theta^e$.

LB

Angular momentum zero & 3 body & cats . . . ,

leftovers

Milnor

Heis.

Darboux & Cartan rank.

$\theta \wedge d\theta^r$

Extremes: integrable

contact or ps-contact.

in terms of solns to Pfaff eqns

maxima: $n-1$ dim'l

min. $[\frac{n}{2}]$ dim'l.