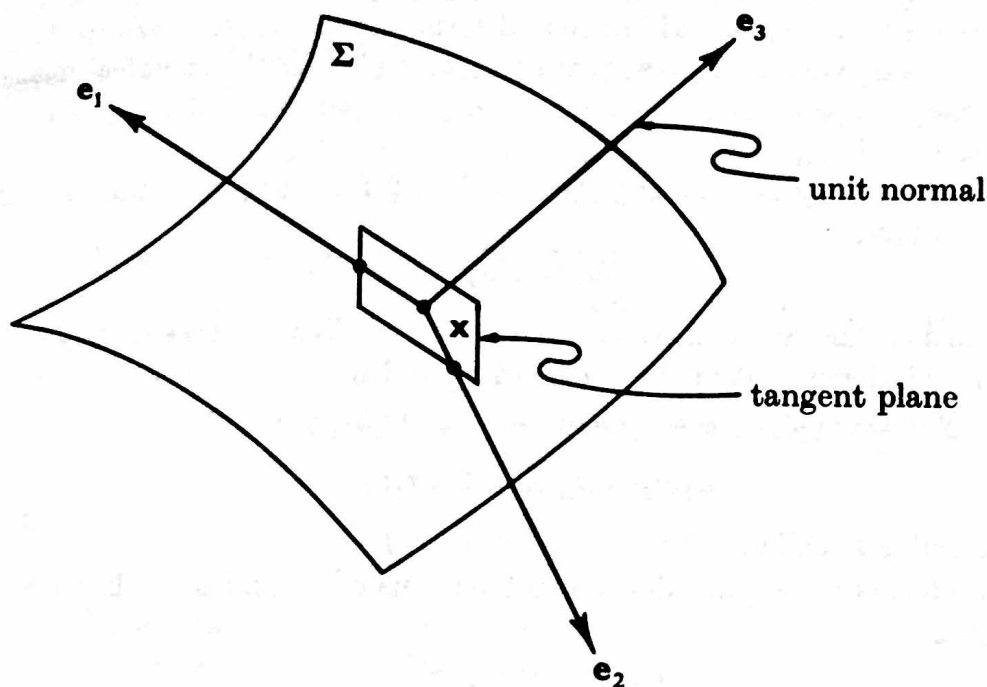


Since \mathbf{x} is on the surface, $d\mathbf{x}$ must lie in the tangent plane, $\sigma_3 = 0$:

$$d\mathbf{x} = \sigma_1 \mathbf{e}_1 + \sigma_2 \mathbf{e}_2.$$

It is clear that the two-form $\sigma_1 \sigma_2$ represents the element of area of Σ .



We exploit the skew-symmetry of Ω by writing

$$\Omega = \begin{pmatrix} 0 & \varpi & -\omega_1 \\ -\varpi & 0 & -\omega_2 \\ \omega_1 & \omega_2 & 0 \end{pmatrix}.$$

The structure and integrability conditions now reduce to

Structure equations

$$\left\{ \begin{array}{l} d\mathbf{x} = \sigma_1 \mathbf{e}_1 + \sigma_2 \mathbf{e}_2 \\ d\mathbf{e}_1 = \varpi \mathbf{e}_2 - \omega_1 \mathbf{e}_3 \\ d\mathbf{e}_2 = -\varpi \mathbf{e}_1 - \omega_2 \mathbf{e}_3 \\ d\mathbf{e}_3 = \omega_1 \mathbf{e}_1 + \omega_2 \mathbf{e}_2 \end{array} \right.$$

Integrability conditions

$$\left\{ \begin{array}{l} d\sigma_1 = \varpi \sigma_2 \\ d\sigma_2 = -\varpi \sigma_1 \\ \sigma_1 \omega_1 + \sigma_2 \omega_2 = 0 \\ d\varpi + \omega_1 \omega_2 = 0 \\ d\omega_1 = \varpi \omega_2 \\ d\omega_2 = -\varpi \omega_1 \end{array} \right.$$