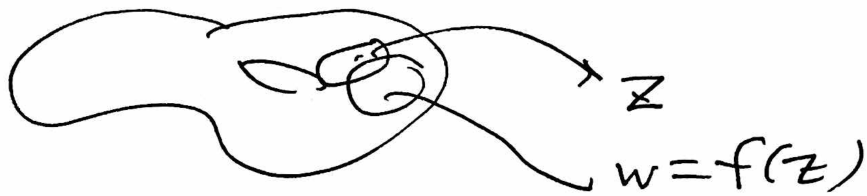
Roots of Differential Geometry R1

Topology.

Gauss: Surfaces in $\mathbb{R}^3$
from pt. of view of bug,
spherical, hyperbolic 2-d
geometry;

$\Rightarrow$ Riemann: Rieman surfaces



overlaps:

$$\begin{array}{c} U \subset \mathbb{R}^2 \simeq \mathbb{C} \\ \downarrow \text{holo} \\ V \subset \mathbb{R}^2 \simeq \mathbb{C} \end{array}$$

Riemannian geometry:
constant sectional Riem mfds
S^n, $\mathbb{R}^n$, $\mathbb{H}^n$.

$\Rightarrow$

R 2

Poincaré: algebraic Topology, notion of manifold getting

Einstein: Riemannian geometry w/ signature (3,1) as foundational to General Relativity.

Takes off!
(E. Cartan, Levi-Civita, ...

$\Sigma^2 \hookrightarrow \mathbb{R}^3$ an smooth embedding.

[Means? .. Def.

then $T_p\Sigma$ a 2-plane.

$\langle\ ,\ \rangle_p : T_p\Sigma \times T_p\Sigma \to \mathbb{R}$.

$\vert$ = Euc inner prod restricted to $T_p\Sigma$.

Eg S^2:

$$ds^2 = \langle\, , \rangle = d\varphi^2 + \sin^2\varphi \, d\theta^2 .$$

$$= \theta_1^2 + \theta_2^2 .$$

Q: When are two surfaces $\Sigma_1, \Sigma_2 \subset \mathbb{R}^3$

a) ambiently isometric?

b) isometric?

Meaning...?

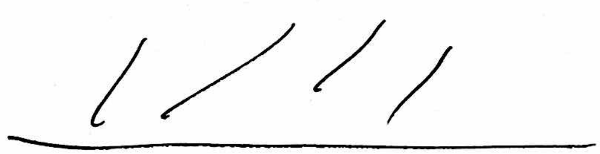
R 4

On to Riem. geometry.

<u>Def</u>. a Riem mfld is a mfld M w a smoothly varying inner product $\langle \cdot , \cdot \rangle_m$ $m \in M$ on each $T_m M$.

<u>Eg</u>. 1: ~~Σa~~ $M^n \subset \mathbb{R}^N$ as before.

<u>Eg</u> 2 $\mathbb{H}^2$: $\dfrac{dx^2 + dy^2}{y^2}$ on $y > 0$.



Poinc. upper half plane.

Loc expression:

$$\langle \ , \ \rangle = ds^2 .$$

$$ds^2 = \sum_{i,j} g_{ij}(x) dx^i dx^j$$

$$= \sum_{a=1}^{n} (\theta^a)^2 \qquad \theta^a \in \Omega^1(U)$$

R5.

w/ $g_{ij}(x)$ pos. def metric

$\exists$ of : : Prop Every smooth mfd M admits a Riem metric

(Partition of unity

Lorentzian,

$\mathbb{R}^{3,1}$: $dx^2+dy^2+dz^2-c^2dt^2$.

and $\sum \varepsilon_a (dx^a)^2$.

$dx^4 = c\,dt$ so $x^4 = ct$.

<u>G. R.</u>: $ds^2 = \sum_{\mu,\nu=1}^{4} g_{\mu\nu}\,dx^\mu dx^\nu$

$g_{\mu\nu}$ sym, invertible signature $(3,1)$