Structure of class,
for now.

some lecture
some worked HW or other problems
some student presentation.
always open for questions

main texts: Flanders
Guillemin Pollack
Sjamaar.

alternatives Burke I & II
Lee.
etc etc.

oh. weekly HW:
try to do it all.
hand in <u>one</u> at least every Tu
be ready to present one " week.

postings: my web site. for class.
HW: in a Dropbox.

1. <u>1-forms</u>

a one-form on the plane is an expression of the form

$$\alpha = P(x,y)\,dx + Q(x,y)\,dy$$

<u>meaning</u>: integrand for line integrals.

$c: I \to \mathbb{R}^2$. $c(t) = (x(t), y(t))$.

$$c^*\alpha = P(x(t),y(t))\frac{dx}{dt}\,dt + Q(x(t),y(t))\frac{dy}{dt}\,dt.$$

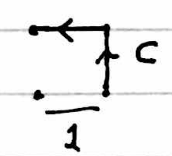
$$\int_c \alpha = \int_I c^*\alpha.$$

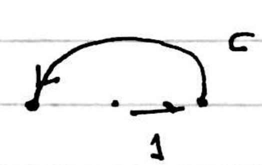
I an interval.

<u>indep of how c is parameterized</u>.

Someone: tell me: what that means?

eg: $\int x\,dy = ?$





Basic properties

$$\int_c \alpha = -\int_{-c} \alpha$$

who's $-c$?

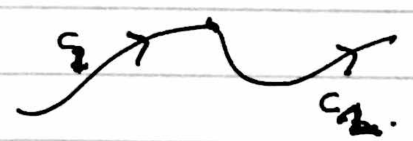
if $I = [0,1]$ $\quad -c(t) = c(1-t)$.

$$\int_{c_2 * c_1} \alpha = \int_{c_1} \alpha + \int_{c_2} \alpha.$$

$$\int_c A\alpha + B\beta = A\int_c \alpha + B\int_c \beta.$$

$A, B \in \mathbb{R}$

What's $c_1 * c_2$?



$A\alpha + B\beta$?

$f : \mathbb{R}^2 \to \mathbb{R}$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy .$$

<u>Prop</u> $\displaystyle\int_c df = f(B) - f(A)$

if $c : [a,b] \to \mathbb{R}^2$
$c(a) = A$, $c(b) = B$.

<u>Pf</u> $c^* df = \frac{d}{dt}(f \circ c)(t)\, dt$.

Now use F Thm Calc

$\therefore$ if c is closed $\therefore A = B$

$\displaystyle\int_c df = 0.$

write $\oint$ for closed curves integrals

<u>Prop</u> If $\displaystyle\int_c \alpha = 0$ $\forall$ closed curves c then $\alpha = df$ for some $f : \mathbb{R}^2 \to \mathbb{R}$.

Pf... ?

$d: \Omega^k \to \Omega^{k+1}$.

$\Omega^0 =$ fns
$\Omega^1 =$ 1-forms
$\Omega^2 =$ 2-forms
= expressions of forms
$g(x,y)\,dx \wedge dy$.
rule: $dx \wedge dy = -dy \wedge dx$.
= integrands for integrals over regions.

Computation $ddf = 0$.

Poincare lemma: the following are equiv. for $\alpha \in \Omega^1(\mathbb{R}^2)$.

1) $\forall$ closed curves c $\int_c \alpha = 0$.

2) $d\alpha = 0$

3) $\exists f \in \Omega^0$ $\alpha = df$.