

Break out -

$\mathbb{C} \setminus 0$ is a Lie group
under complex multiplication.
Coordinatize $G = \mathbb{C} \setminus 0$ in
polar coordinates:

$$\omega = \frac{dz}{z}$$

$$z = re^{i\theta}$$

Compute the Maurer-Cartan ω
form in polar coordinates

$$\omega = \frac{dz}{z}$$

Recompute it in Cartesian:

$$z = x + iy$$

thereby finding formulae

$$d\theta = \psi(x, y, dx, dy)$$

$$\frac{dr}{r} = \phi(x, y, dx, dy)$$