BREAK-OUT

A smooth one-form

$$\alpha = P(x,y)\,dx + Q(x,y)\,dy$$

has the property that

$$\int_I \alpha = 0$$

for all line segments I parallel to the x-axis

1) Find the general expression for such an α

2) Suppose, in addition, that

$$\oint_{Box} \alpha = \text{Area}(Box)$$

for all rectangles 'Box' whose sides are parallel to the axes & which are oriented counter-clockwise, with 'Area(Box)' = usual area inside the Box.

a) Find a particular α

b) Find all α obeying these two conditions.