MATH 209, MANIFOLDS II, WINTER 2011

Homework Assignment I: One-forms and integration

1. A one-form α on a smooth connected manifold M is called *exact* if and only if there exists a function f such that $\alpha = df$. Prove

- α is exact $\iff \int_\gamma \alpha = 0$ for any smooth closed curve γ.

2. [HAND IN] Let $\alpha = x\,dy$ on the plane $\mathbb{R}^2$ with coordinates (x, y). Let S^1_R denote the circle of radius $R > 0$ centered at the origin, oriented counter clockwise. Evaluate

$$\int_{S^1_R} \alpha.$$

Conclude that α is not exact.

If γ is any closed simple curve in $\mathbb{R}^2$ what is the geometrical meaning of $\int_\gamma x\,dy$?

3. Find three different one-forms α in the plane for which $d\alpha = dx \wedge dy$.

4. Find a function $S = S(x, y)$ such that $xdy = \frac{1}{2}(xdy - ydx) + dS$.

5. Let $\alpha = (x\,dy - ydx)/(x^2 + y^2)$.

a) Show that $d\alpha = 0$.

b) Show that $\int_{S^1_R} \alpha = 2\pi$ where S^1_R is as per problem **2**.

c) Show that in polar coordinates (r, θ) we have $\alpha = d\theta$.

d) Explain why (c) does not contradict problem **1**.