MATH 209, MANIFOLDS II, WINTER 2011

Homework Assignment Week 5: One-forms and integration

1. Verify that

$$s([1,z]) = \frac{1}{\sqrt{1+|z|^2}}(1,z)$$

is a section of the Hopf fibration whose domain U_0 is all of $\mathbb{CP}^1$ but the single point $[0,1]$.

a) Let $\phi : z \mapsto [1,z]$ be the affine chart covering U_0. Compute $\phi^* s^* A$ in the affine chart, where $A = Im\langle\psi, d\psi\rangle$ is the connection for the Hopf fibration described in class and lecture notes.

b) Show that the curvature F of A in the affine chart, which is to say, $\phi^* F$, is equal to $\frac{1}{i}\frac{d\bar{z}\wedge dz}{(1+|z|^2)^2}$

c) Verify that $\int_{\mathbb{CP}^1} F = 2\pi$ by computing the integral of $\phi^* F$ over $\mathbb{C}$.