Manifolds 1 Homework 1

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b) It turns out that N is in fact not diffeomorphic to the tangent bundle of S^1 but the mbius band. As such first we show that the mbius band is diffeomorphic to N and then prove the separate problem of the tangent bundle of S^1 being diffeomorphic to the cylinder.

To show that N is diffeomorphic to the mbius band we define a smooth map from N to $\mathbb{RP}^2$ minus a point. And to do this we define a map from N to $\mathbb{R}^3 \setminus 0$ that is constant on the fibers of the quotient map from $\mathbb{R}^3 \to \mathbb{RP}^2$ which we will denote π. First note that we can write any line in N in the form $ax + by + c = 0$ as long as a and b are not both zero. Now not that $\lambda(ax + by + c = 0) = ax + by + c$. That is to say multiplying this equation by a scaler doesn't change what line it is in the plane. Now we define $\phi : N \to \mathbb{R}^3 \setminus 0$ by $ax + by + c = 0 \mapsto \frac{(}{a}, b, c)$. As sated above this function's inverse image on linear subspaces of $\mathbb{R}^3$ map to a single line(which is a point in N) showing that ϕ is constant on the fibers of π. It follows that this descends to a smooth map $\pi \circ \phi : N \to \mathbb{RP}^2$. This map is almost surjective only missing the point that is the linear span of $(0, 0, 1)$ in $\mathbb{RP}^2$. It is otherwise bijective and invertible exactly because any scaler multiple of a line is the same line and scaler leave you in the same linear span (point) in $\mathbb{RP}^2$. This map shows that N is diffeomorphic to $\mathbb{RP}^2$ minus a point. Though I will not prove it here a little bit of algebraic topology on plane models of the projective plane and the mbius strip will yeah that the projective plane minus a point is homeomorphic (and in fact diffeomorphic) to the mbius band without a boundary. $\square$