VECTOR FIELDS HOMEWORK assigments.

1. Let X be a vector field on a manifold, and p a point of the manifold and suppose that $X(p) \neq 0$. Prove that there is another vector field Y on the manifold such that the equality $[X, Y] = X$ holds in a neighborhood of p.

2. Suppose that X is a vector field on a manifold M , and p is a point of the manifold. Use the coordinate definitions of vector fields ("An n-component vector $X^1, \ldots, X^n$ depending on the choice of coordinates which transforms under changes of variables like... ") to prove:

(i) If $X(p) = 0$ then the first derivative of X at p is well-defined and can be viewed as a linear map $DX_p : T_pM \to T_pM$.

(ii) If $X(p) \neq 0$ then the first derivative of X at p, depends on the choice of coordinates and hence is not well-defined.

3. $Gl(2, \mathbb{R})$ acts on $\mathbb{RP}^1$. Write the action as $(g, [x, y]) \mapsto g \cdot [x, y] = [ax + by, cx + dy]$. By the standard affine chart let us mean the map $x \mapsto [x, 1]$ or its inverse.

A) Verify that this action is the standard linear fractional transformation upon being viewed in the standard affine chart.

B) The associated infinitesimal action is the linear map $\sigma : gl(2, \mathbb{R}) \to \chi(\mathbb{RP}^2)$ which sends a 2 by 2 matrix $\xi \in gl(2, \mathbb{R})$ to the vector field $\sigma(\xi)$ on $\mathbb{RP}^1$, the infinitesimal generator of this action: $\sigma(\xi)([x, y]) = \frac{d}{dt}|_{t=0} exp(t\xi) \cdot [x, y]$. Write out the infinitesimal generator map *explicitly* by writing out, rel. to the standard affine coordinate, the four vector fields corresponding to the image of the standard basis E_1, E_2, E_3, E_4 of $gl(2, \mathbb{R})$ under σ. Your answers will be of the form $\sigma(E_i) = f_i(x) \frac{\partial}{\partial x}$, with explicit functions $f_i(x)$, $i = 1, 2, 3, 4$.