HW 3.

3.1 A). Look up and state the definition of a manifold being "parallelizable".

3.1 B) Show that S^1 is parallelizable.

3.1 C) Show that $O(n)$ is paralleliable. Hint: Use the diffeos $O(n) \to O(n)$ given by $L_A(g) = Ag$ and the previous HW where you computed tangent spaces of $O(n)$.

3.2. The standard cone $x^2 + y^2 = z^2$ is a two-dimensional surface C which is singular at the origin $0 = (0, 0, 0)$. Compute the dimension of its cotangent space , defined as $\mathfrak{m}_p/\mathfrak{m}_p^2$, for $p = 0$. Then recompute for $p \neq 0, p \in C$.

3.3. An oriented line is a line endowed with a sense of direction. **Prove:** the space of oriented lines in $\mathbb{R}^{n+1}$ is diffeomorphic to the tangent bundle to the n-sphere. (Hint: the direction is a unit vector $\vec{u}$, hence a point of the sphere. Now on each line ℓ, mark the point $P \in \ell$ closest to the origin. You have defined a map $\ell \mapsto (u, P)$ ...)