MATH 208 - HW # 4 - Corrections

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Problem 1. View x as the affine coordinate for $\mathbb{RP}^1$ so as to identify $\mathbb{RP}^1$ with $\mathbb{R} \cup \{\infty\}$. Show that the vector field $\frac{\partial}{\partial x}$ on the open set $\mathbb{R} \subset \mathbb{RP}^1$ extends to a smooth vector field on all of $\mathbb{RP}^1$. Does this vector field vanish at ∞?

Solution 1. Using the overlap map $y = \frac{1}{x}$ we can say:

$$\frac{\partial}{\partial x} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\partial}{\partial y} = -\frac{1}{x^2}\frac{\partial}{\partial y} = -y^2\frac{\partial}{\partial y}$$

As $x \to \infty$ the line $[x, 1]$ tends to $[1, 0]$. Consequently $y \to 0$ in this direction telling us that:

$$\frac{\partial}{\partial x} \to 0 \quad \text{as} \quad x \to \infty$$

Therefore, $\frac{\partial}{\partial x}$ extends to a smooth vector field on $\mathbb{R} \cup \{\infty\}$ that vanishes at ∞.