MATH 208 - HW # 1

Nathan Marianovsky
October 4, 2017

Problem 1. Show that the set of points $(x, y) \in \mathbb{R}^2$ satisfying $xy = 0$ does not constitute a topological manifold.

Solution 1. We first note that all points satisfying the given relation belong to either the x-axis, y-axis, or both for the case of the origin. This means that at any point besides the origin we are locally homeomorphic to $\mathbb{R}$. We want to show it is impossible to find a homeomorphism that will take any neighborhood of the origin into $\mathbb{R}$ since it is homeomorphic to $\mathbb{R}^2$. Recall that connectedness is a property preserved by homeomorphisms. Thus, let I_1, I_2, I_3 and I_4 represent the four distinct intervals found in a neighborhood of the origin that intersect at a single point. Under a homeomorphism these intervals must remain distinct and intersect a single point. Unfortunately, this is impossible in $\mathbb{R}$ for more than two intervals because it would contradict the distinct assumption.