Math 208 = Manifolds I . FINAL. Due Friday December 8, 2017

1. M is a compact manifold without boundary. Prove that any atlas for M must contain at least two charts.

2. Let M be a manifold and S^1 the circle. Construct a non-vanishing vector field on the manifold $M \times S^1$.

The next three problems are linked. In them, $\mathcal{L}_V$ denotes the Lie derivative with respect to the vector field V. Use problems 3 and 4 to do 5.

3. Let α be a one-form on a manifold and V a vector field. Then $\alpha \odot \alpha$ is a covariant symmetric two-tensor which we can write as $\alpha \otimes \alpha$. Prove that $\mathcal{L}_V(\alpha \odot \alpha) = 2(\mathcal{L}_V \alpha) \odot \alpha$.

4. Let f be a smooth function on a manifold and V a vector field. Prove that $\mathcal{L}_V df = d(\mathcal{L}_V f)$.

5. Let $g = dx^2 + dy^2$ denote the standard Euclidean metric on the plane. Calculate the subspace of all vector fields V on the plane such that $\mathcal{L}_V g = 0$.

[Hint: this is a 3 dimensional space]