2 True /False. [20 pts.] Credit requires a reason to support your answer. A reason could be a sketch of a proof, an exemplary figure, a counterexample, the statement of a theorem, or reference to a HW.

a) Inside any rhombus one can inscribe a circle.

True

\[ \Delta ABC \text{ and } \Delta ADC \text{ are isosceles, } \Delta BAC \cong \Delta BCA \text{ and } \angle DAC \cong \angle DCA, \]

By SSS \( \Delta ABC \cong \Delta ADC \) and \( \Delta ABD \cong \Delta ACD \). Thus \( \Delta BAC \cong \Delta DAC \cong \Delta BCA \cong \Delta DCA \) and \( \angle ABD \cong \angle DBC \cong \angle ADB \cong \angle BDC \).

This makes AC and BD angle bisectors with intersection O. Like triangle, the intersection is the center of the triangle.\( \text{point } O \), the center of the circle, is the intersection of the 3 angle bisectors of the triangle.\( \text{Point } O, \text{ the center of the circle, is the intersection of the 3 angle bisectors of the triangle.} \)

b) The intersection of the three angle bisectors and the three perpendicular bisectors of a triangle coincide, if that triangle is isosceles.

False; consider the case of an isosceles triangle with a large interior angle inscribed in a circle.

\[ \text{Point } O, \text{ the center of the circle, is the intersection of the 3 bisectors of the triangle as seen in homework. The 3 bisectors of the triangle is the center of the circle inscribed in the triangle.} \]

c) A line \( l \) parallel to the base AB of a triangle ABC cuts the triangle into two parts. If \( l \cap AC \) is the midpoint \( M \) of \( AC \) then these two parts have equal areas.

False; consider a right triangle with legs of 4 and 5.

\[ \text{This a right triangle by Pythagorean, } \]

\[ 3^2 + 4^2 = 5^2 \]

The area is \( \frac{1}{2} (3)(4) = 6 \)

d) Among all triangles which have two sides of length 4 and 5 (in some units) the 3:4:5 triangle has maximal area.

False; consider a right triangle with legs of 4 and 5.

As a right triangle \[ C = \sqrt{16+25} = \sqrt{41}, \]

The area is \( \frac{1}{2} (4)(5) = 10 \)

\[ 6 < 10 \]