1. Solutions, trials, and tribulations on HW2.

The problem which I graded asked "Prove that the points on the angle bisector you constructed in problem 2 of HW1 are EQUIDISTANT from $\ell$ and $m$. (Hint: You must be able to DEFINE the distance between a point and a line. For this see the previous two problems of this HW, just above.)"

If you received less than 6 points on this problem then it is almost certain that you were either solving another problem or did not understand what I was asking you to do. A dead give away for me was that if I saw that you used SAS in your proof. The proof requires ASA.

Here is a restatement of the problem. Show that if $E$ is any point on the angle bisector then $d(E, m) = d(E, \ell)$. The distance $d(P, \ell)$ between a point $P$ and a line $\ell$ were defined and investigated in two earlier problems where you were asked to shown that this distance is realized as the length of the line segment made by dropping the perpendicular from the point $P$ to the line $\ell$.

Here is the picture which fits the problem:

Here is a complete solution.

Let $E$ be a random point on the bisector. Drop a perpendicular from $E$ to ray $m$. Denote this ‘dropped’ line segment $EF$. Thus $F \in m$ and $EF$ is perpendicular to ray $m$. Similarly drop a perpendicular from $E$ to ray $\ell$. Call this ‘dropped’ line segment $EG$. Thus $G \in \ell$ and $EG$ is perpendicular to ray $\ell$.

Since $d(E, m) = EF$ and $d(E, \ell) = EG$, we must show that: $EF \equiv EG$.

**Proof.** Look at triangles $AEF$ and $AEG$ where $A$ is the vertex of the angle made by the two rays. These triangles have shared side $AE$. Their angles are congruent since the angles at $E$ and at $F$ are both right (90 degrees) and since, by assumption ray $AG$ bisects the given angles so that angles at $A$ in both triangles are equal (equal to $\frac{1}{2}$ the given angle). So by ASA the two triangles are congruent. Thus their respective parts $EF$ and $EG$ are congruent and hence of the same distance.

QED