BACKGROUND, TERMINOLOGY, NOTATION.

In this problem ‘rotation’ means a rotation of Euclidean 3-space $\mathbb{R}^3$ which fixes the origin, and thus defines an orientation preserving isometry of the standard unit sphere. Every rotation with the exception of the identity has a unique axis of rotation - the axis about which it rotates points. That axis can be specified by a point $u$ on the sphere. The fixed points of $R$ on the sphere are then $u$ and $-u$.

Let $N$ denote the north pole $(0,0,1)$ and $P$ the point on the unit sphere for which $x = y = z$ and $x > 0$.

The tangent space to the sphere at a point $u$ is the 2-plane orthogonal to $u$ and is written $T_u S^2$.

PROBLEM. [with 5 parts]
A) Describe a rotation $R_0$ which $N$ to $P$ and whose axis of rotation lies on the xy plane. Describe $R$ both by saying precisely (i) what its axis and angle of rotation is, and also by (ii) writing out the matrix for $R$ relative to the standard basis.

B) For $u$ a point on the sphere write $G_u$ for the subgroup of all rotations whose axis is $u$. If $g \in G_N$ and $h \in G_P$ show that (i) $R = R_0 g$ and $hR_0$ both take $N$ to $P$.

C) Find a rotation matrix $R$ taking $N$ to $p$ while at the same time taking the tangent vector $e_1 = (1,0,0) \in T_N S^2$ to the unit tangent vector $v_1 \in T_p S^2$ of the form $(a,b,0)$ with $b > 0$.

D) True or False: The rotation matrix $R$ of problem part (C) is unique.

E) Let $L$ denote the ‘line of latitude’ 10 degrees, which is to say the set of points on the unit sphere for which $z = \cos(10^\circ)$. True or False: there exists a rotation matrix whose axis of rotation lies on $L$ and which takes $N$ to $P$. 