19A Practice Final

March 10, 2018

For problems 1-3, let $f(x) = 2xe^{-x^2}$.

Problem 1

(a) Draw the graph of f

(b) Are there any critical points?

(c) If so, identify which (if any) correspond to local min/max's

Problem 2

Find the equation for the tangent line at the point $(0, f(0))$.

Problem 3

Express

$$F(x) = \int_0^x f(t) \, dt$$

explicitly by evaluating the integral.

Problem 4

Identify (all!) the local extrema of the function $g(x) = x\sin(x) + \cos(x)$.

Problem 5 Box problem

Suppose we are putting together a cardboard box with no top, with dimensions length (l), width (w), and height (h) so that $l = 2w$, and whose volume is $V = lwh = 50$. What is the minimum amount of cardboard needed to achieve this?

Problem 6

Apply Newton's method to the function $f(x) = x^2 - x - 1$ to approximate the golden ratio $\Phi = \frac{1+\sqrt{5}}{2}$. Note that Φ satisfies $\Phi^2 = \Phi + 1$.

Note: approximate Φ up to at least two non-integer values.

Problem 7

Let

$$G(x) = \int_0^x \sin(t^2)\ dt.$$

Compute $\frac{d}{dx}G(x) = G'(x)$.

Problem 8

Suppose a population P with intial population $P_0 := P(0) = 100$ is modeled by the differential equation

$$\frac{d}{dt}P(t) = P'(t) = \frac{1}{1+t^2}P(t).$$

Find $P(t)$.

Problem 9

Maximize the product xy when x and y are constrained to the ellipse given by the equation

$$x^2 - xy + y^2 = 1.$$

(Think about whether or not your answer makes intuitive sense.)