

Final Review Problems. LAST WEEK of...

Problem 1. TO DO AGAIN. Question of continuity involving the “squeezing lemma”

Example: A function  $f(x)$  satisfies  $1 \leq f(x) \leq e^x$ . What can you say about the continuity of  $f$  at  $x = 0$ ?

Problem 2. DONE. A question involving Newton iteration and using the linear (tangent line) approximation.

Example: set up the Newton iteration scheme for finding the square root of some given integer.

Problem 3. DONE. [3/5] . A question involving the fundamental theorem of calculus.

Example: differentiate  $G(x) = \int_1^{x^2} s \log(s) ds$

Problem 4. DO TODAY, 3/15. A question on L'Hopital's rule. ( You could also use 2nd order Taylor expansions.)

Example. Compute  $\lim_{t \rightarrow 0^+} \frac{e^t + e^{-t} - 2}{t^2}$ .

Problem 5. DONE: 3/9. A question involving implicit differentiation.

Example: A point  $(x, y)$  is constrained to move along the ellipse given by the equation  $x^2 - xy + 4y^2 = 4$ .

A) Find an expression for  $dy/dx$  as a function of  $x$  and  $y$ .

B) Use your expression to find the two points on the ellipse at which the tangent lines are parallel to the line  $y = x$ .

PROBLEM 6. TO DO: A simple optimization problem. (Boxes; Sums fixed. )

PROBLEM 7. \*\*\* NOT ON TEST \*\*\* problem on exponential growth or decay.

PROBLEM 8. DONE: A fundamental theorem of calculus question of the type given last week. I draw the graph of a function  $f$  which is the sum of step functions and piecewise linear functions. You draw the graph of  $\int_0^x f(s) ds$ .

Example: Graph  $\int_0^x f(s) ds$ , for  $x \geq 0$ , when

$f(x) = 1$  for  $0 < x < 2$ ,  $f(x) = 0$  for  $2 < x < 4$  and  $f(x) = -x + 4$  for  $x > 4$ .

PROBLEM 9. TO BE DONE. A graphing problem.

Example: Graph some functions of the form  $f(x) = Ax^a + Bx^{-b}$  for  $x > 0$ . For various exponents  $a, b$ , find values of the parameters  $A, B$  which guarantee a critical point exists in the positive quadrant (I.e the region  $x > 0$  and  $y > 0$  of the plane. Locate this critical point.

PROBLEM 10. TO DO AGAIN. A problem involving the method of bisection \*\*\* and / OR the intermediate value theorem.