Final Review Problems. Dates in brackets preceding problem refer to the likely date in class when the problem will be covered.

[3/2] 1. Question of continuity invoving the "squeezing lemma' .

Example: A function $f(x)$ satisfies $1 \leq f(x) \leq e^x$. What can you say about the continuity of f at $x = 0$?

2. A question involving Newton iteration and using the linear (tangent line) approximation.

Example: set up the Newtion iteration scheme for finding the square root of some given integer.

[3/5] 3. A question involving the fundamental theorem of calculus.

Example: differentiate $G(x) = \int_1^{x^2} s \log(s) ds$

[3/2] 4. A question on L'Hopital's rule. (You could also use 2nd order Taylor expansions.)

Example. Compute $\lim_{t \to 0+} \frac{e^t + e^{-t} - 2}{t^2}$.

[3/9] 5. A question involving implicit differentiation.

Example: A point (x, y) is constrained to move along the ellipse given by the equation $x^2 - xy + 4y^2 = 4$.

A) Find an expression for dy/dx as a function of x and y.

B) Use your expression to find the two points on the ellipse at which the tangent lines are parallel to the line $y = x$.

6. A simple optimization problem. (Boxes; Sums fixed.)

7. A problem involving exponential growth or decay.

[3/ 7] 8. Another fundamental theorem of calculus question of the type given last week: I draw for you the graph of a combination of step functions and piecewise linear functions. You are to draw its integral ("moving sum"', indefinite integral) from 0 to x, x now being the independet variable.

Example: Graph $\int_0^x f(s) ds$, for $x \geq 0$, when

$f(x) = 1$ for $0 < x < 2$, $f(x) = 0$ for $2 < x < 4$ and $f(x) = -x + 4$ for $x > 4$.

9. A graphing problem.

Example: Graph some functions of the form $f(x) = Ax^a + Bx^{-b}$ for $x > 0$. For various exponents a, b, find values of the parameters A, B which guarantee a critical point exists in the positive quadrant (I.e the region $x > 0$ and $y > 0$ of the plane. Locate this critical point.

10. A problem involving the method of bisection.

Possible extra credit problems:

11. A problem involving geometric series.

12. A problem involving the size of infinite sets.