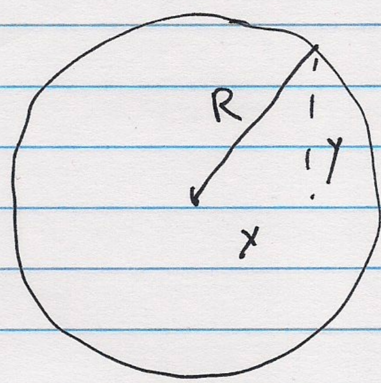
Final Review :

#5. "A problem involving implicit differentiation"

<u>First</u> : recall circle eqn: $x^2 + y^2 = R^2$ ← radius constant



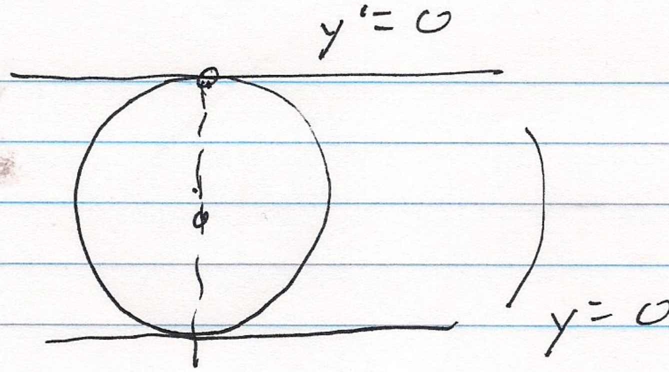
Differentiate eqn, viewing y as a function of x. Get:

$$2x + 2yy' = 0 \quad \text{since } \frac{dR}{dx} = 0.$$

so $yy' = -x$

$$y' = \frac{-x}{y}$$

Reality checks: $x = 0 \Rightarrow (x,y) = (0, \pm R)$ top of circle: slope 0. ✓



at top & bottom of circle.

2nd reality check, Solve:

$$y^2 = R^2 - x^2$$

$$y = +\sqrt{R^2 - x^2} = y(x)$$

or

$$-\sqrt{R^2 - x^2}$$

Compute $\frac{dy}{dx}$ for either one.

Find: $\frac{dy}{dx} = -\frac{x}{y(x)}$.

Divide:

$$\frac{2x-y}{x-8y} = y'$$

B). Use (A) to find the points on the ellipse at which the tangent line is parallel to the line $y=x$.

Sol'n: y' = <u>slope</u> of tangent.

~~a line~~: $y=x$ has slope 1
so a line is tangent to $y=x$
iff it has slope 1
(form $y = x+b$).

So <u>set</u> $y'=1$. above:

$$\frac{2x-y}{x-8y} = 1$$

or
$$2x-y = x-8y$$

or $x = -7y$; $y = -\frac{1}{7}x$.

The point (x,y) must also lie on the ellipse:

$$x^2 - xy + 4y^2 = 4$$

lets plug in: $x = -7y$

$$\Rightarrow 49y^2 + 7y \cdot y + 4y^2 = 4$$

$$60y^2 = 4$$

$$y^2 = \frac{4}{60} = \frac{1}{15}$$

$$y = \pm\sqrt{\frac{1}{15}}$$

From earlier $x = -7y$

Solutions: $\left(-7\sqrt{\frac{1}{15}}, \sqrt{1/15}\right), \left(+7\sqrt{\frac{1}{15}}, -\sqrt{\frac{1}{15}}\right)$

Picture