Final Review; Problem 4

A L'Hopital problems.

i) $\frac{e^t + e^{-t} - 2}{t^2}$ vs ii) $\frac{e^t + e^{-t}}{t^2}$.

iii) vs $\frac{e^t - e^{-t}}{t^2}$.

as $t \to 0$.

Could (i) & (ii) both have finite limits? as $t \to 0$

No! Their difference is $\frac{-2}{t^2}$ which goes to ∞

&

$\lim_{t \to 0} (\mathcal{F}(t) + \mathcal{G}(t)) = \lim_{t \to 0} \mathcal{F}(t) + \lim_{t \to 0} \mathcal{G}(t)$.

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i) $\frac{f(t)}{g(t)}$; $f(t) = e^t + e^{-t} - 2$,

$\lim_{t \to 0} f(t) = e^0 + e^{-0} - 2$

$= 1 + 1 - 2 = 0$.

$\lim_{t \to 0} g(t) = \lim_{t \to 0} t^2 = 0^2 = 0$

i) $0/0$

what is the form of ii)?

What is the limit of (ii)?

i) ~~g~~ ctd
L'Hop if f, g ~~cts~~ differentiable at $t=0$
& $f(0) = g(0) = 0$ then
$$\lim_{t \to 0} \frac{f(t)}{g(t)} = \frac{f'(0)}{g'(0)}$$
provided $f'(0)$, $g'(0)$ not both zero!

$f'(t) = e^t - e^{-t}$
$f'(0) = 1 - 1 = 0$

$g'(t) = 2t$
$g'(0) = 2 \cdot 0 = 0$

Again of form $0/0$.

Use L'Hop again.

$f''(t) = e^t + e^{-t}$
$f''(0) = 1 + 1 = 2$

$g''(t) = 2$
$g''(0) = 2,$

$$\Rightarrow \frac{f''(0)}{g''(0)} = \frac{2}{2} = 1$$

Review, Final ; Problem 4

Way 2.

~~Ex~~ $e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + \frac{t^4}{4!} \cdots$

where $n! = n((n-1)!)$

& $1! = 1$

so

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ etc.

[Why?

$f(t) = e^t$ is the unique function

such that

1) $\frac{df}{dt} = f$

2) $f(0) = 1$.

Suppose:

$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + \ldots$

then

$f'(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + \ldots$

&

$f(0) = a_0$

So: $a_0 = 1$

$f(t) = f'(t) \Rightarrow a_1 = a_0$

$2a_2 = a_1$

$3a_3 = a_2$
$4a_4 = a_3$

$\rightarrow \quad a_0 = a_1 = 1$
$2a_2 = 1 \Rightarrow a_2 = \frac{1}{2}.$
$3a_3 = a_2 \Rightarrow a_3 = \frac{1}{3} \cdot \frac{1}{2}. = \frac{1}{3!}$
$4a_4 = a_3 \Rightarrow a_4 = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{4!}$
etc

So:
$$e^{-t} = 1 + (-t) + \frac{(-t)^2}{2} + \frac{(-t)^3}{3!} + \cdots$$

$$e^{t} = 1 + t + \frac{t^2}{2} + \cdots$$
$$+ \; e^{-t} = 1 - t + \frac{t^2}{2} + \cdots$$

$\rightarrow$

$$e^t + e^{-t} = 2 + t^2 + O(t^4)$$ ← terms involving t^4

$$e^t + e^{-t} - 2 = t^2 + \cdots$$

$$\frac{e^t + e^{-t} - 2}{t^2} = \frac{t^2 + O(t^4)}{t^2} = 1 + O(t^2)$$
$$\rightarrow 1.$$