Soln to an 'FTC' + chain rule problem

Compute: $G'(x)$ where $G(x) = \int_0^{x^2} s\ln(s)\,ds$.

($\ln = \log_e$.)

<u>Sol'n</u>. Use F.T.C.:

1) if $F'(x) = f(x)$ then $\int_a^b f(s)\,ds = F(b) - F(a)$

so: Let $F(x)$ be <u>any</u> anti-derivative of $x\ln(x) = f(x)$;

* → You <u>do not</u> need to find F, just trust it's there!!

so → $\boxed{F'(x) = x\ln x.}$

Then

$$\int_0^x s\ln s\,ds = F(x) - F(0)$$

So:

$$G(x) = F(x^2) - F(0)$$

&

$$G'(x) = F'(x^2)(x^2)' - 0.$$

$$= f(x^2)\cdot 2x$$

but $F'(x) = x\ln x = f(x)$!

$$= x^2\ln(x^2)\cdot 2x$$

$$= 2x^3\ln(x^2).$$

if you like, clean this up

since $\ln(x^2) = 2\ln x$

so also: $G'(x) = \cancel{4}2x^3\ln(x^2)$

$$\boxed{G'(x) = 4x^3\ln(x).}$$