Problem 10

Intermediate Value, Bisection.

$$P(x) = x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x - 5.$$

Theorem there is at least one positive root x_*

that is

$$P(x_*) = 0, \quad x_* > 0$$

Pf: $P(0) = -5$

$\lim_{x \to \infty} P(x) = +\infty$ so for

$$x >> 1 \qquad P(x) > 0.$$

By I.V.T there is an x_*,
$0 < x_* < \infty$ with $P(x_*) = 0$.

Note: true no matter what
a_4, a_3, a_2, a_1
are. **Question:**

What conditions on $P'(x)$ might guarantee that there is only one positive root. ?

· cf Descartes rule of signs

Count sign changes in coefficients

+ + − + − .. −

The the number of positive roots is even or odd according to the # of sign changes of coefficients of $p(x)$.