SECRET CODE :

NAME:

SECTION and TA's NAME :

Midterm. Calculus (Math 19A). W 2018. D.

1. [25 pts]
2. [25]
3. [20]
4. [20]
5. [10]

Sum.

Show your work to get credit.

1. [25] Use the linear approximation to estimate $(31)^{1/5}$. Write the answer as a mixed fraction (such as $7 + 1/4$). Well... $1^5 = 1$. $2^5 = 32$, so $(31)^{1/5}$ is close to 2, a bit less.

The linear approximation states:

$$f(x_0 + h) \simeq f(x_0) + f'(x_0)h$$

For us, $f(x) = x^{1/5}$. $x_0 = 32$. $h = -1$, since

$31 = 32 - 1 = 32 + (-1)$.

$$f'(x) = \frac{1}{5} x^{\frac{1}{5}-1} = \frac{1}{5} x^{-4/5}$$

$$f'(32) = \frac{1}{5} (32)^{-4/5} = \frac{1}{5} \frac{(32)^{1/5}}{32} = \frac{1}{5} \frac{2}{32} = \dots$$

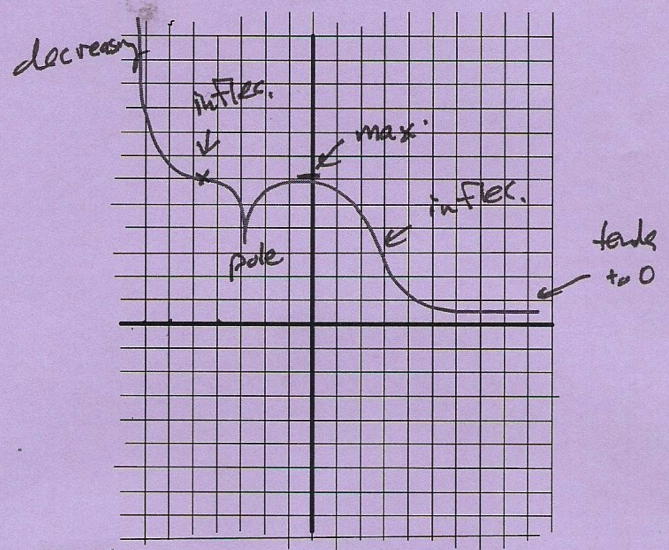
$$= \frac{1}{5} \frac{1}{16} = \frac{1}{80}.$$

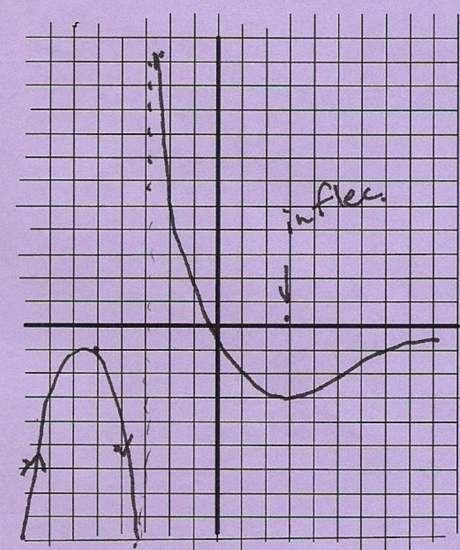
So:

$$\sqrt[5]{31} \simeq \sqrt[5]{32} + \frac{1}{80}(-1) = 2 - \frac{1}{80}.$$

2. [25] The graph of a function is shown.

(A)On the blank graph paper to its right sketch the graph of the derivative of this function.





(B) Using standard interval notation, describe the intervals over which the derivative is negative. (*Assume the dark lines to be the coordinate axes and parallel lines to be spaced 1 unit apart.* As an example of standard interval notation to describe the set of x's such that $-1.2 < x \leq 1$ we write $(-1.2, 1]$.)

Negative deriv $\Leftrightarrow$ function decreasing

$(-8, -2.9)$, $(0, \infty)$

3. [20] Find a polynomial $q(x)$ such that the function $y(x) = e^{q(x)}$ satisfies $y'(x) = (x^2+1)y(x)$.

$y'(x) = q'(x)\, e^{q(x)} = q'(x) y(x)$ by chain rule

$= q'(x) y(x)$

so $q'(x) = x^2+1$. Try: $q(x) = Ax^3 + Bx^2 + Cx + D$.

$q'(x) = 3Ax^2 + 2Bx + C \stackrel{?}{=} x^2+1$

$\Rightarrow 3A = 1,\ 2B = 0;\ C = 1$

$q(x) = \frac{1}{3}x^3 + x + \text{const.}$ works.

4. [20] Find the constant c so that the derivative of $x^4 + cx^2 + 10$ at $x = 1$ is 10.

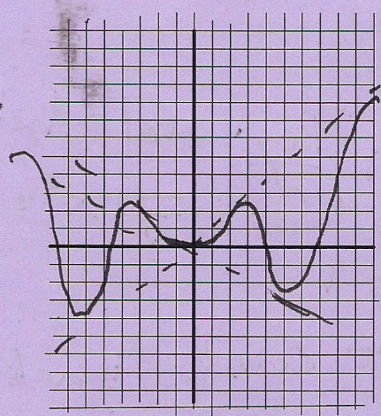
$(x^4 + cx^2 + 10)' = 4x^3 + 2cx$.

@ $x = 1$ get: $4(1)^3 + 2c \cdot 1 = 4 + 2c$

require: $4 + 2c = 10$.

so $c = 3$. $(4 + 6 = 10 \checkmark)$

5. [10] A) Sketch the graph of the function $g(t) = t\sin(t)$.



B) Either find a t with $1 < t < 11$ and for which $\frac{dg}{dt}|_t \geq t+1$ or show that there is no such t in the given interval $(1, 11)$.

$\frac{d}{dt}(t \sin t) = \sin t + t \cos t$ by prod rule.

for all t, $\sin t \leq 1$
$\cos t \leq 1$.

so for $t > 0$

$\sin t + t \cos t \leq 1 + t \cdot 1 = 1+t.$

can you ever get "=" ? Would need: $\cos t = 1$. But whenever $\cos t = 1$ have $\sin t = 0$. $\Rightarrow$ Impossible!

<u>No such</u> t for $t > 0$, let alone $1 < t < 11$