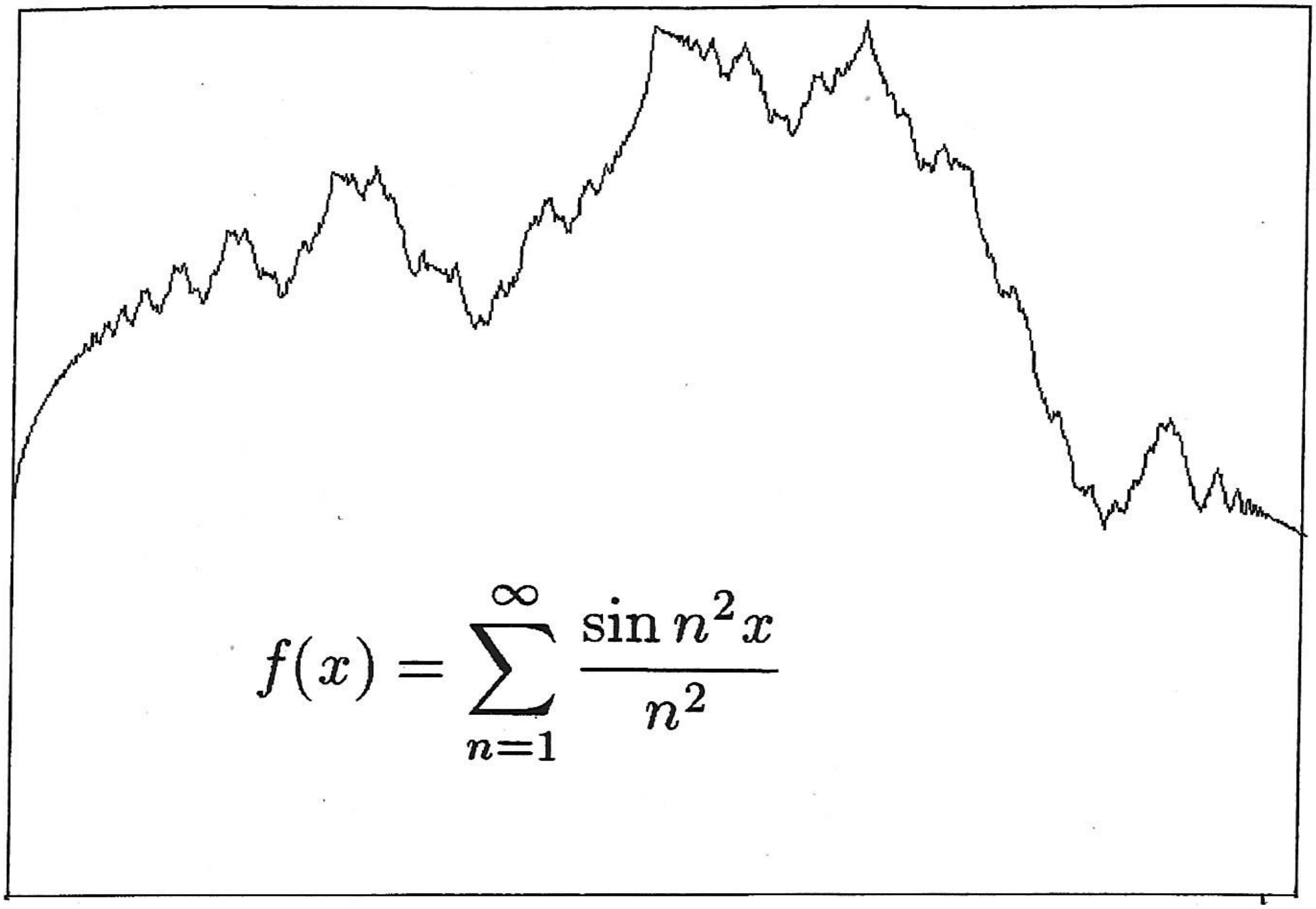
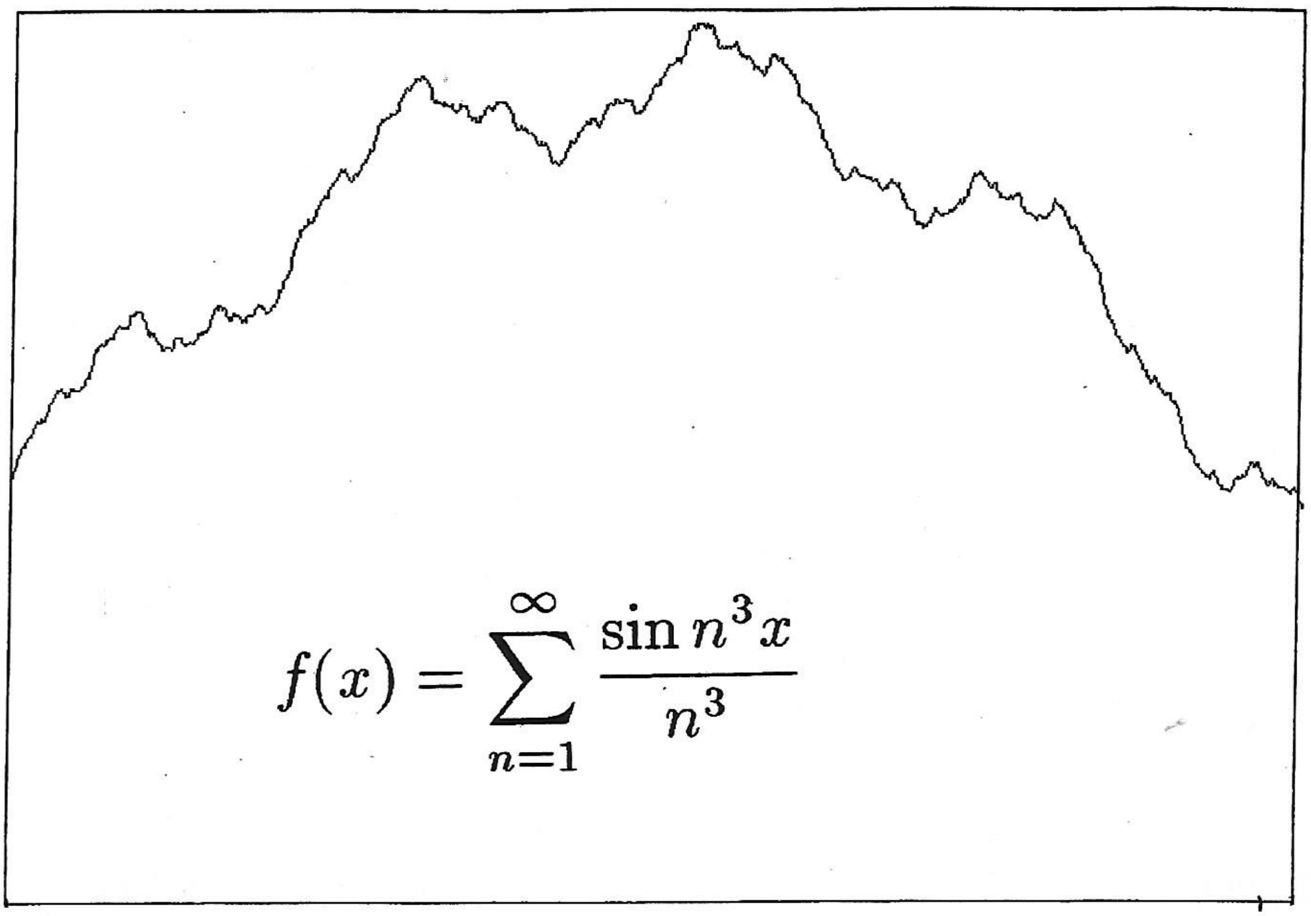
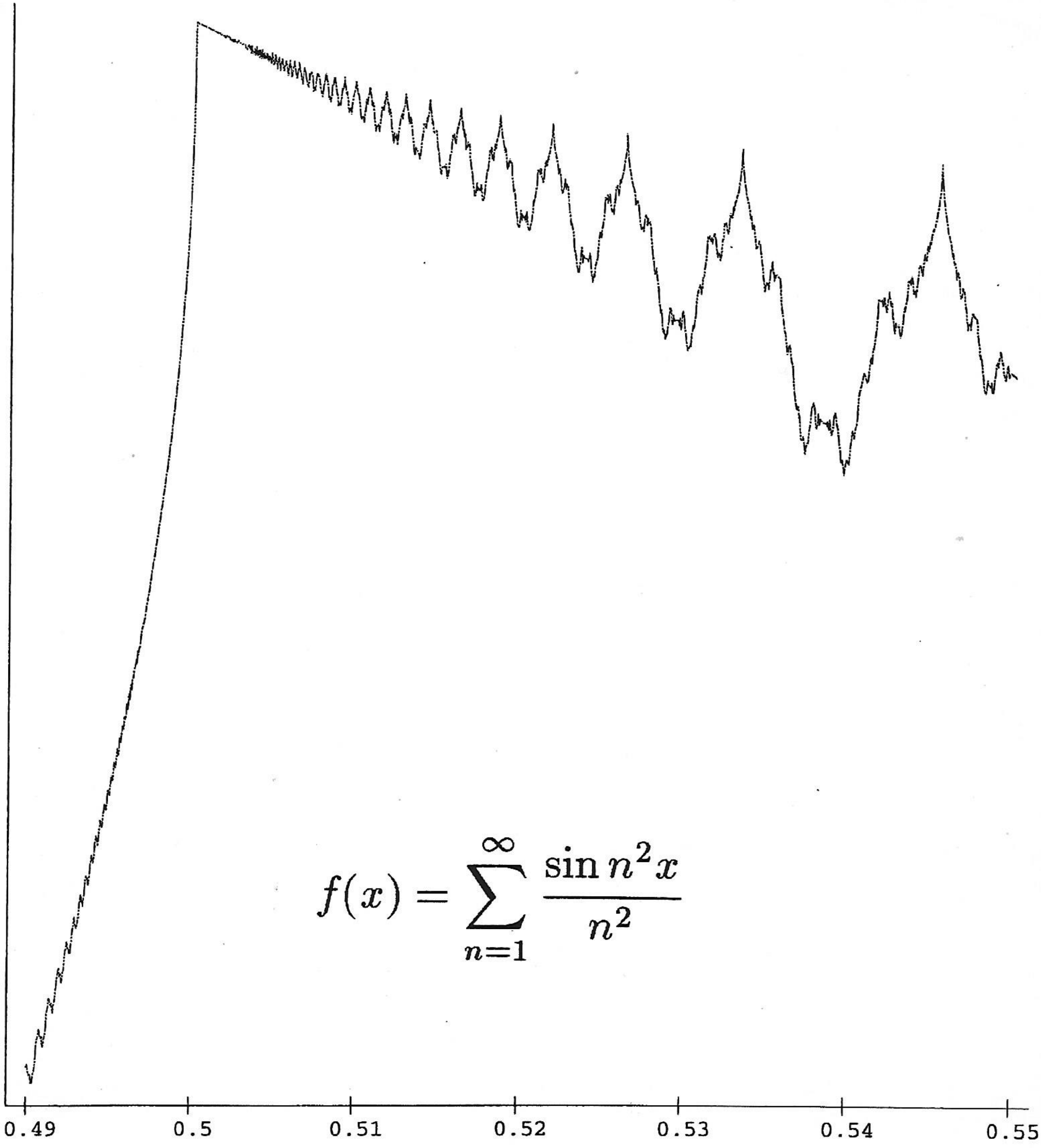


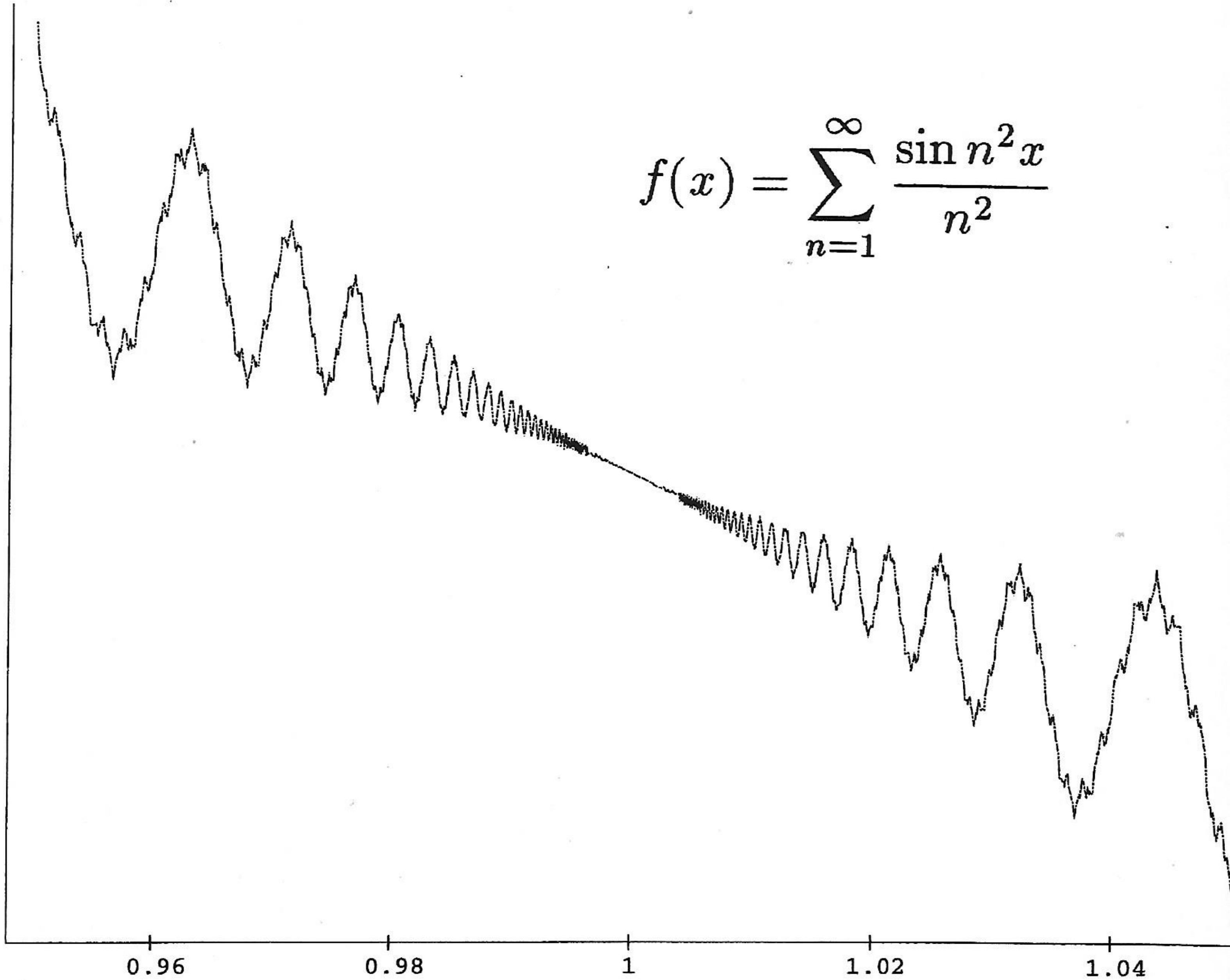
$$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^2}$$



$$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^3}$$

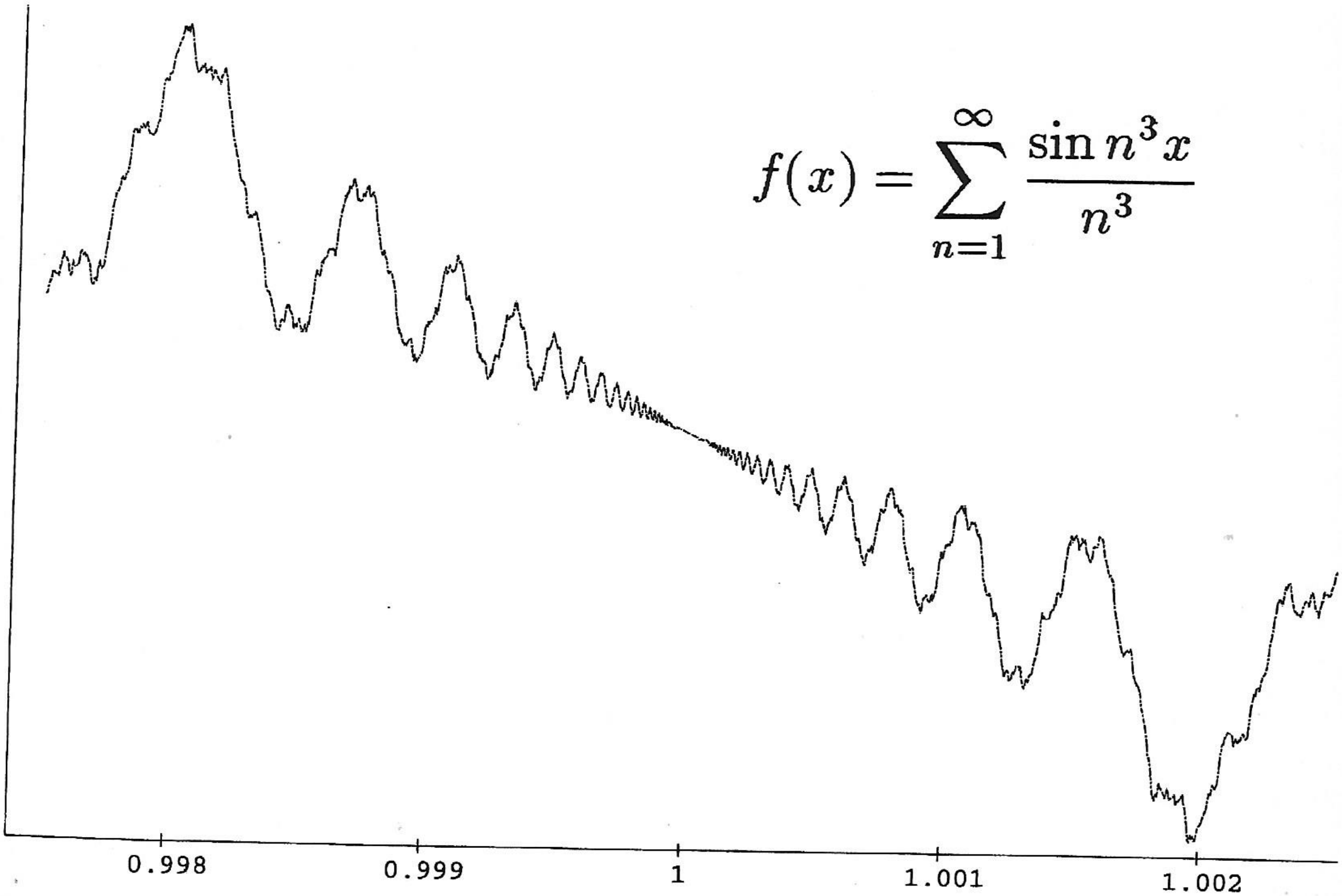


$$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^2}$$



quadratic series: chirp term $= x^{3/2} g(x^{-1})$,

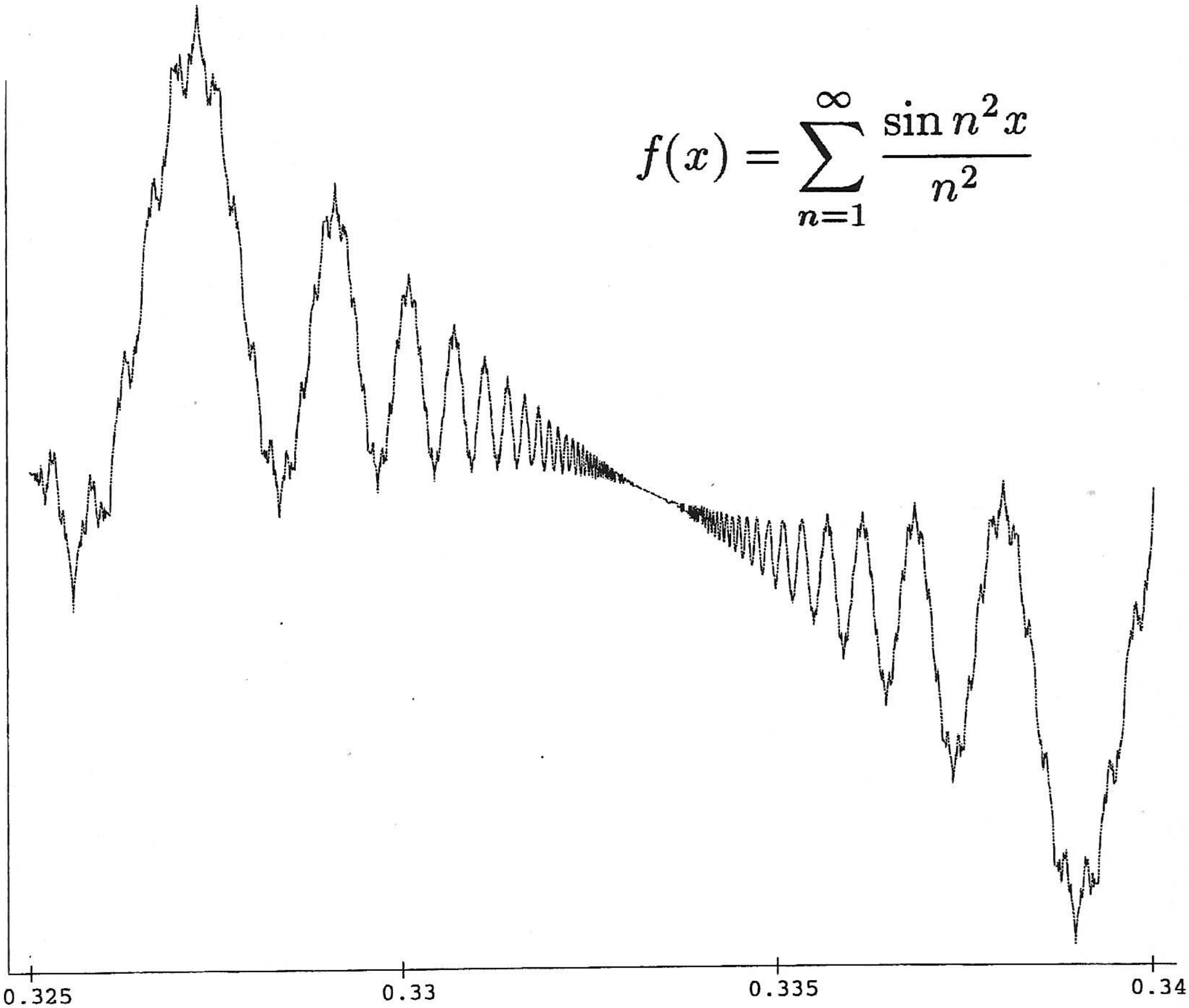
$$\text{where} \qquad g(u) = \sum_{m=1}^{\infty} \frac{\sin m^2 (c_1 u + c_0)}{m^2}$$



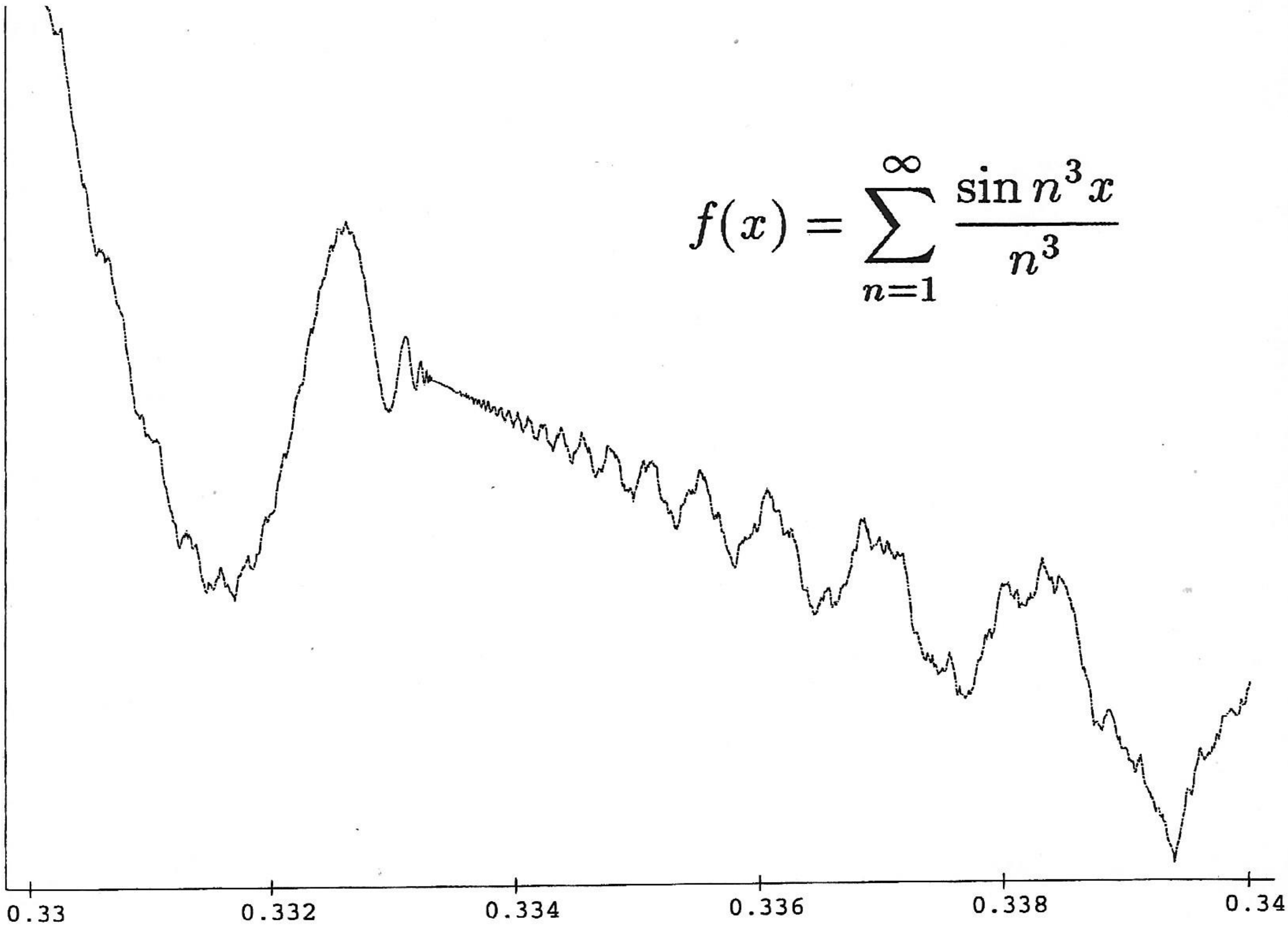
$$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^3}$$

cubic series: $\quad$ chirp term $= x^{5/4} g(x^{-1/2})$,

where $\quad g(u) = \sum_{m=1}^{\infty} \frac{\sin m^{3/2}(c_1 u + c_0)}{m^{7/4}}$



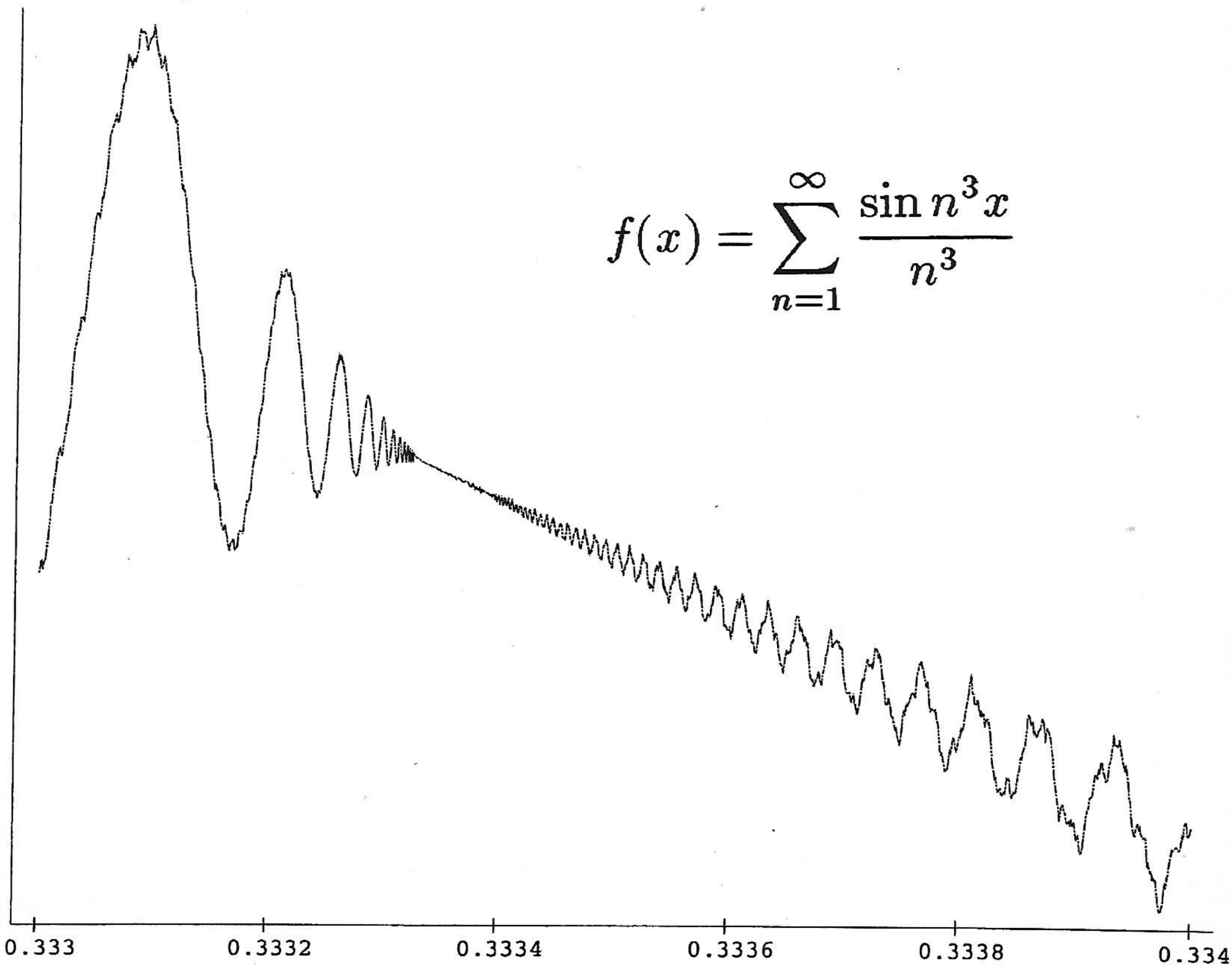
$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^2}$
0.325
0.33
0.335
0.34

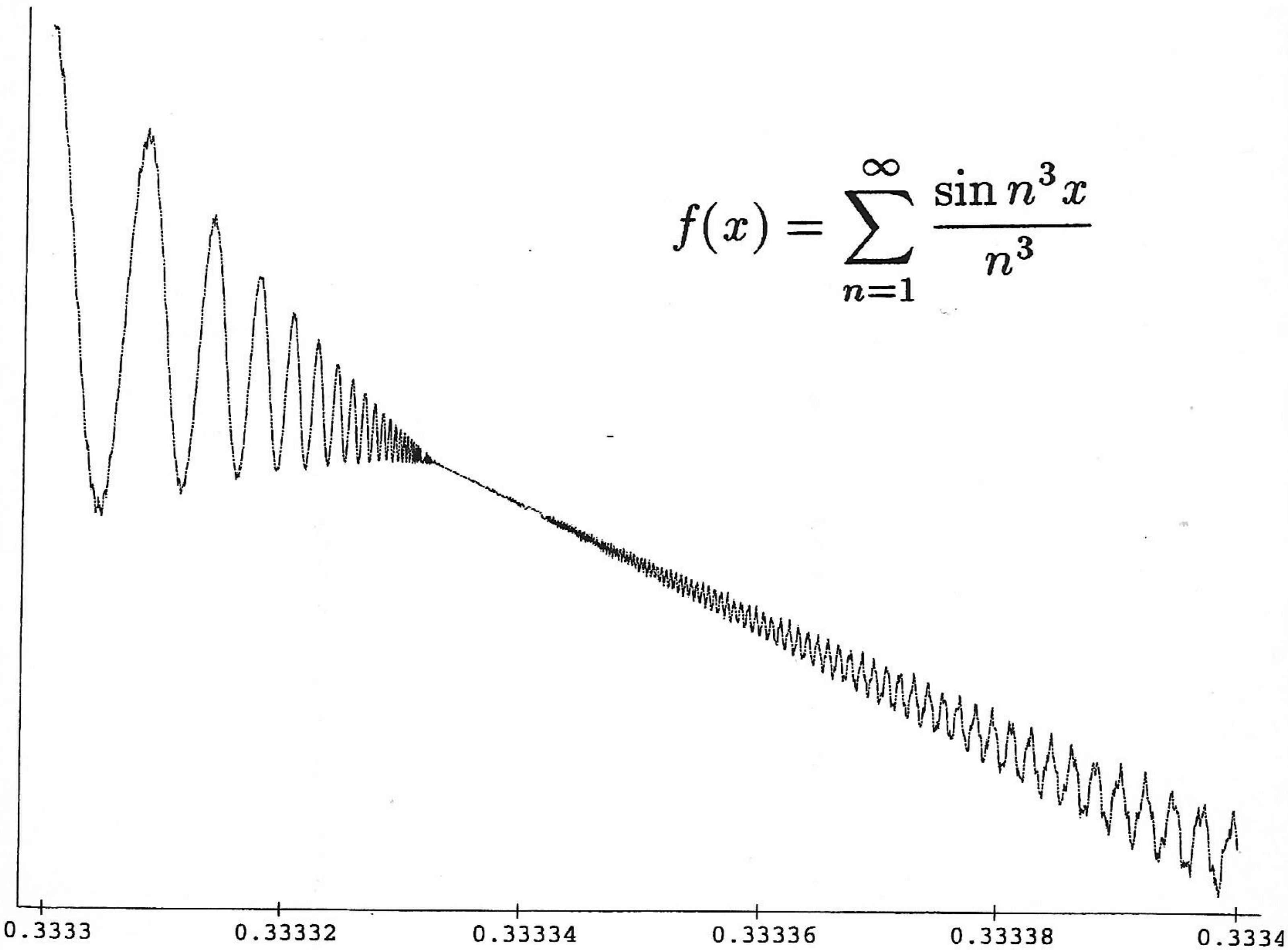


$$g(u) = \sum_{m=1}^{\infty} \frac{\sin m^{3/2}(c_1 u + c_0)}{m^{7/4}}$$

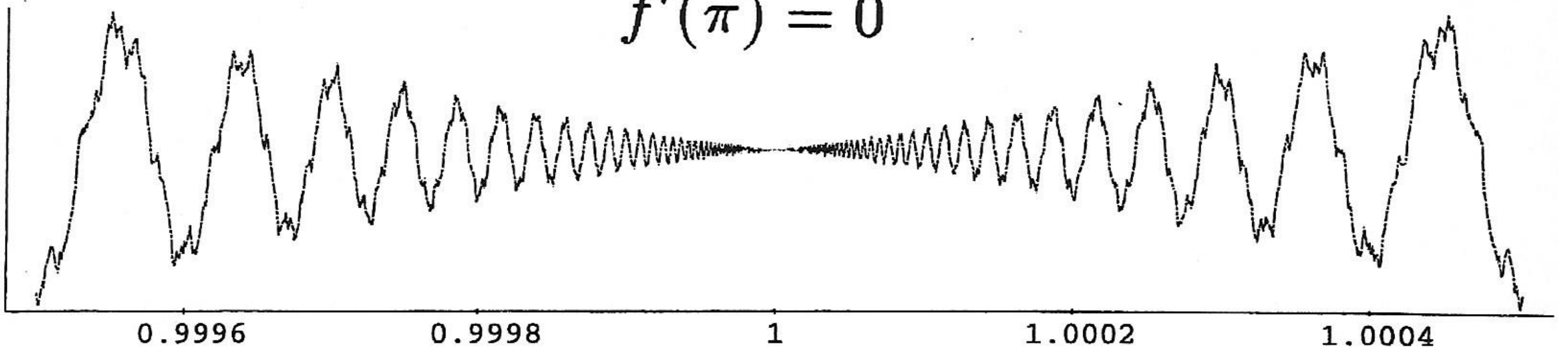
left chirp term: $m \equiv 1 \bmod 6$

right chirp term: $m \equiv 5 \bmod 6$


$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^3}$
0.333
0.3332
0.3334
0.3336
0.3338
0.334

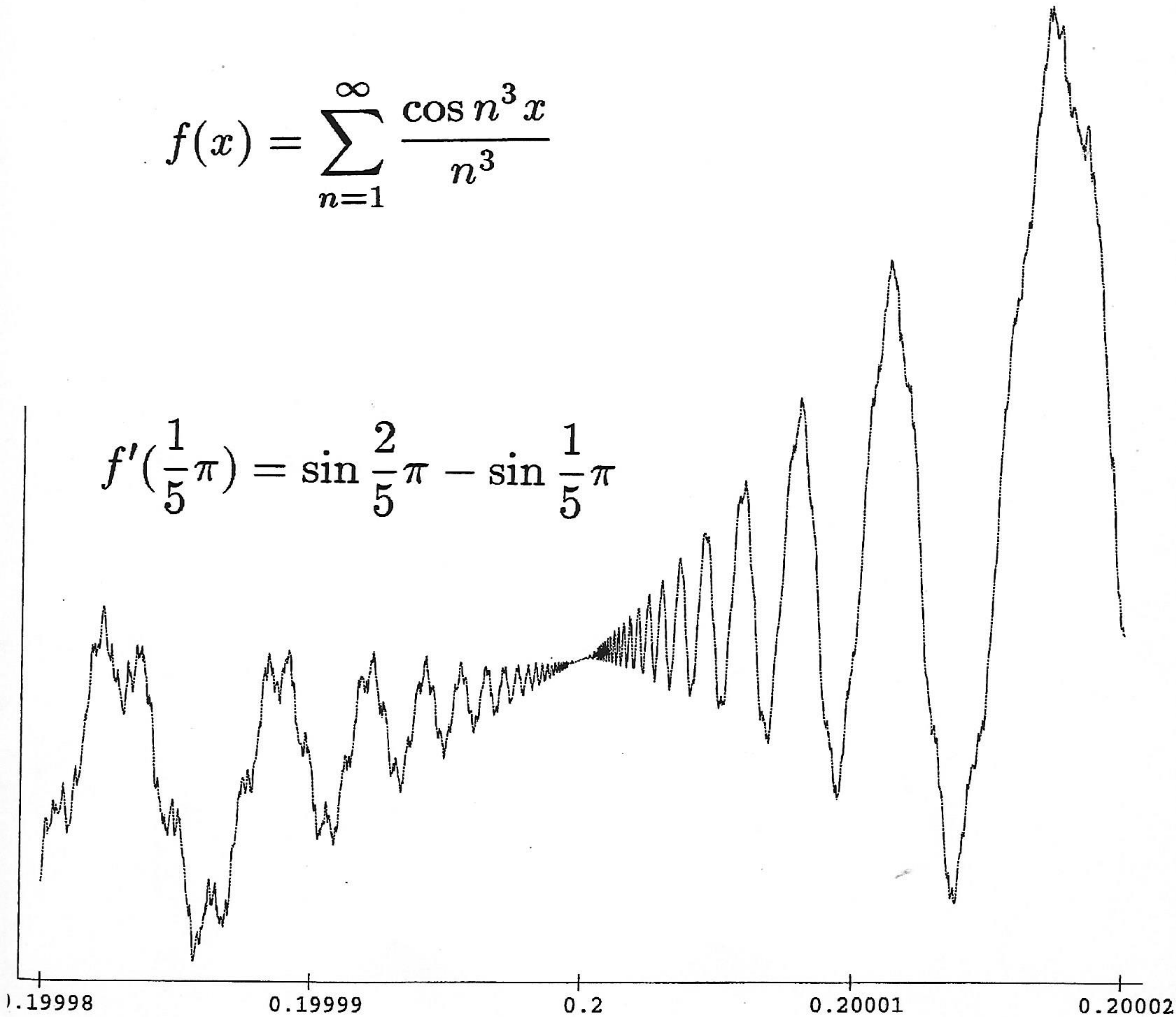

$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^3}$
0.3333
0.33332
0.33334
0.33336
0.33338
0.3334

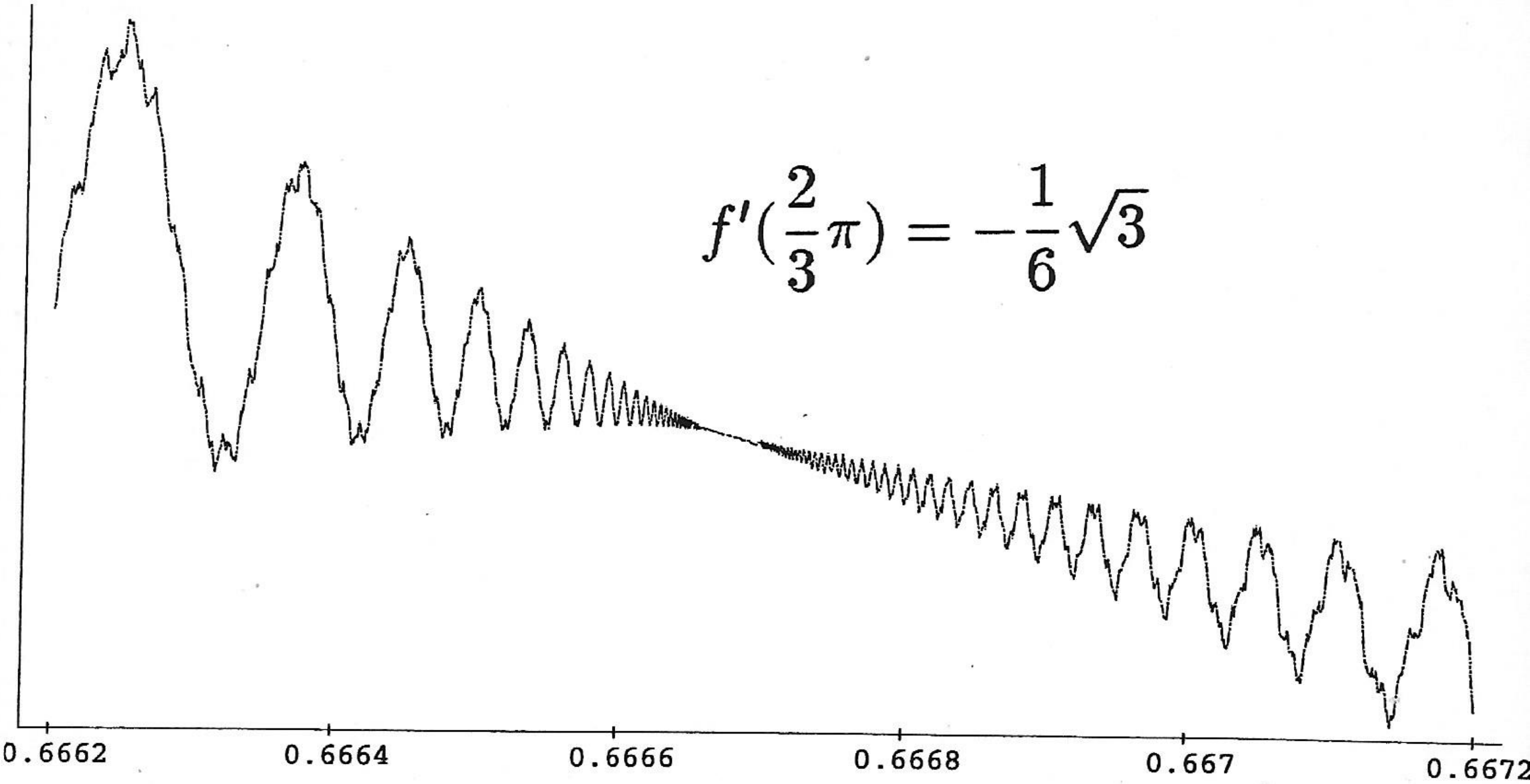
$$f'(\pi) = 0$$



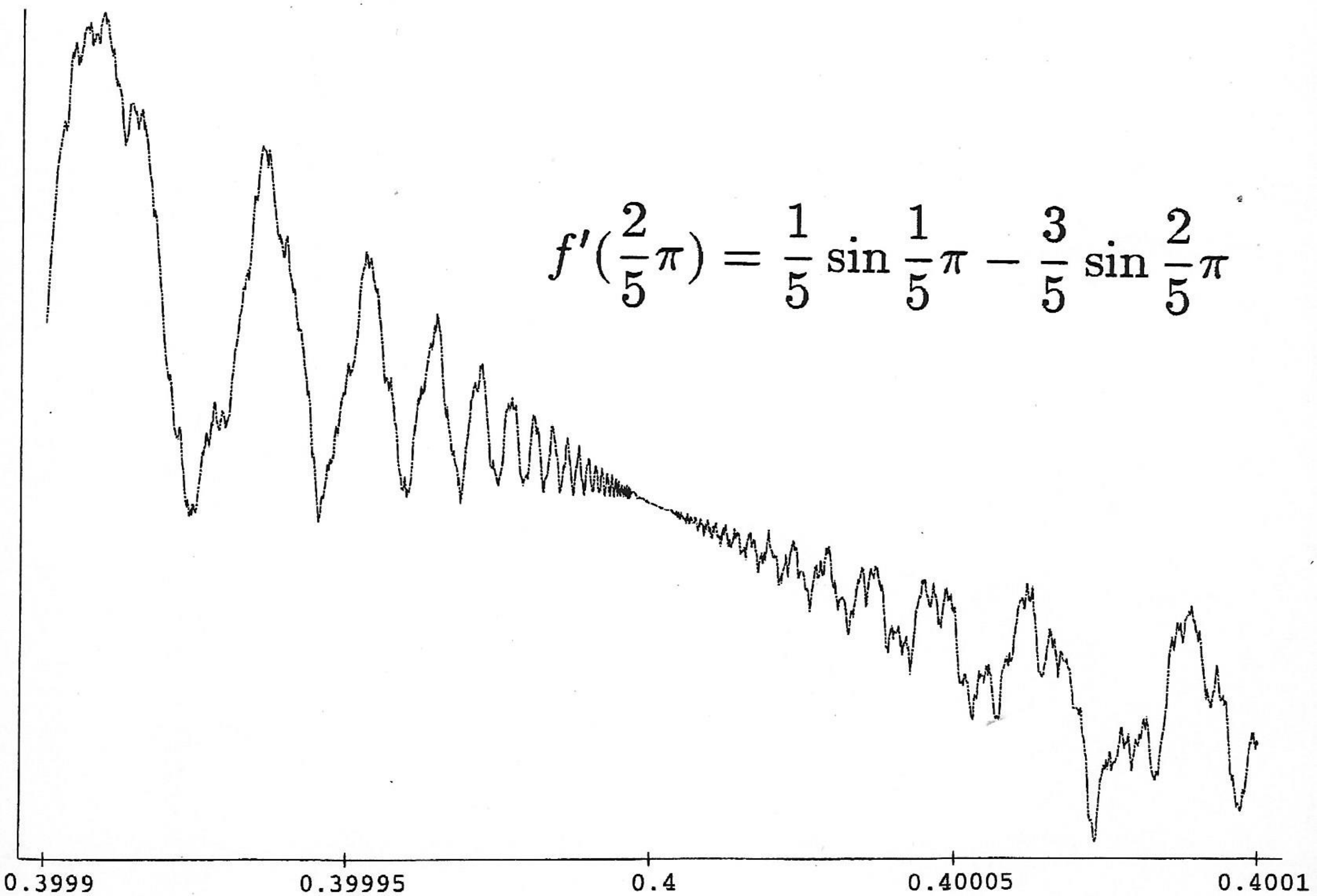
$$f(x) = \sum_{n=1}^{\infty} \frac{\cos n^3 x}{n^3}$$

$$f'(\frac{1}{5}\pi) = \sin \frac{2}{5}\pi - \sin \frac{1}{5}\pi$$

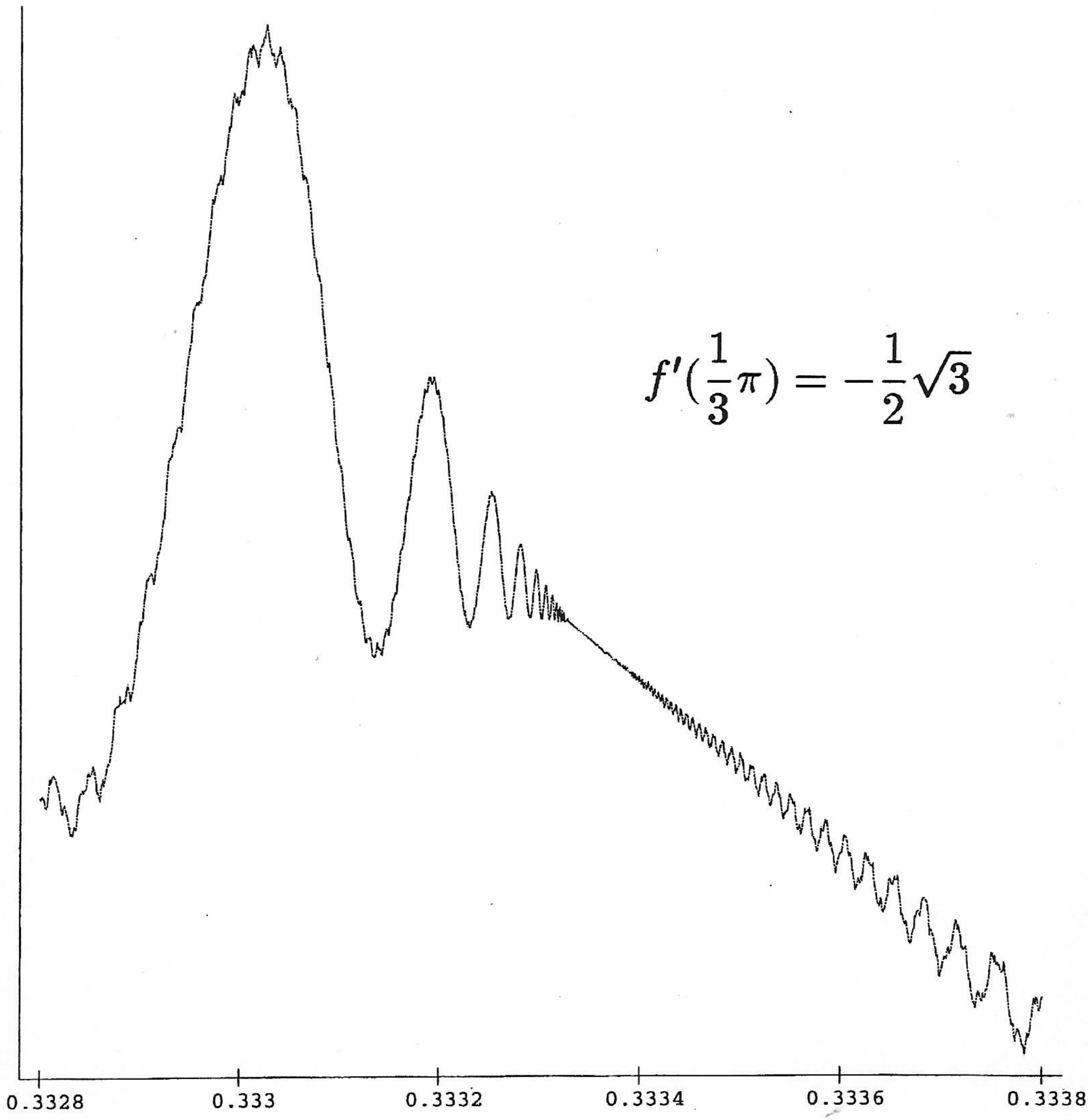




$f'(\frac{2}{3}\pi) = -\frac{1}{6}\sqrt{3}$
0.6662
0.6664
0.6666
0.6668
0.667
0.6672



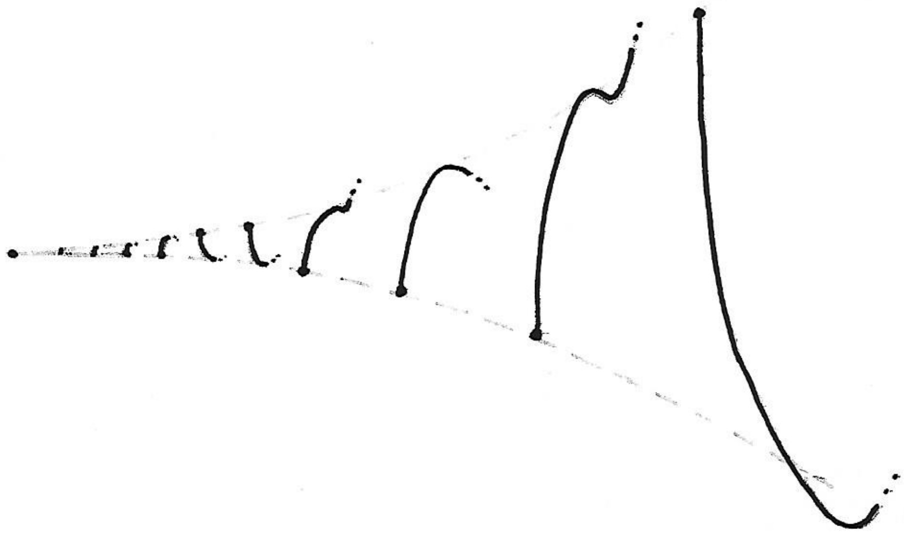
$f'(\frac{2}{5}\pi) = \frac{1}{5}\sin\frac{1}{5}\pi - \frac{3}{5}\sin\frac{2}{5}\pi$
0.3999
0.39995
0.4
0.40005
0.4001



$f'(\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$
0.3328
0.333
0.3332
0.3334
0.3336
0.3338

The left and right derivatives are infinite at $\dfrac{p\pi}{q}$

for all even p and almost all primes $q \equiv 1 \bmod 3$.



$$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^\beta} \quad \text{has no derivative}$$

at almost all irrational points

$$\text{when} \quad \beta \leq \frac{\sqrt{97} - 1}{4} = 2.212\ldots$$

Luther (1986): differentiable nowhere when $\beta \leq 2$

$$\sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^2}$$ has a finite derivative at

$$x = \frac{p\pi}{q}$$ when p and q are both odd.

Otherwise the left or right derivative

(or both) is infinite.

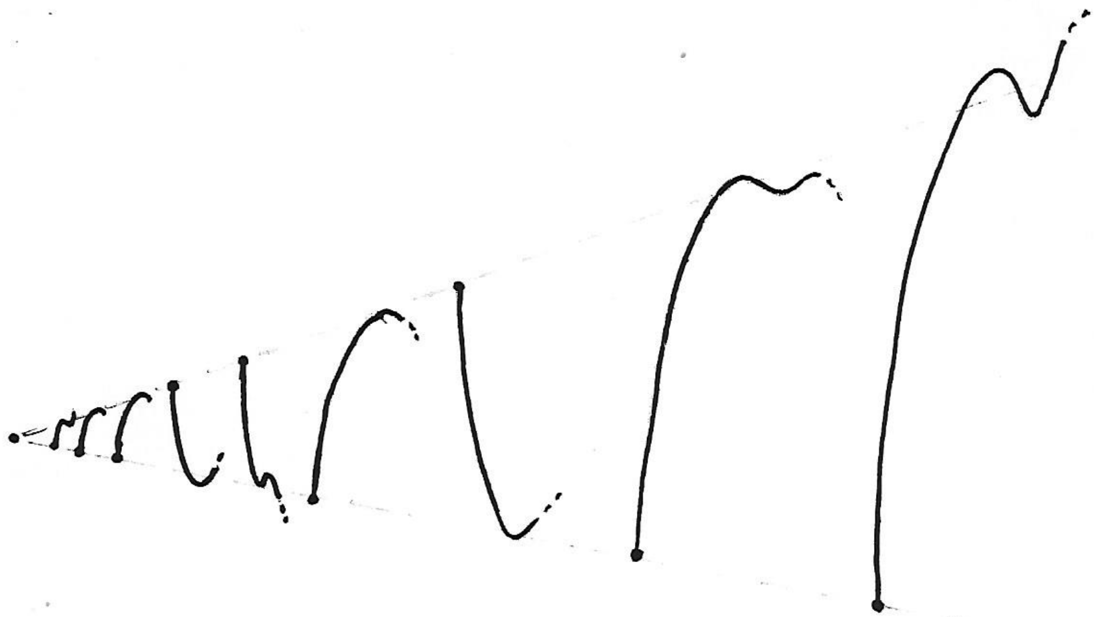
$$\sum_{n=1}^{\infty} \frac{\sin n^3 x}{n^3}$$ has a finite derivative at

$$x = \frac{p\pi}{q}$$ (for p and q relatively prime) if

1) p and q are both odd or

2) q has at least one prime factor $\equiv 2 \bmod 3$

(other than 2) with multiplicity 1.



Hardy: $$f(x) = \sum_{n=1}^{\infty} \frac{\sin n^2 x}{n^\beta}$$ has

no derivative at irrational points

when $\beta \leq \frac{5}{2}$