Cantor

1. Counting the relative sizes of sets.

Put your hands together...

Bijections: 1-1 onto maps.

Defn: Two sets are said to have the same cardinality if one can be put in bijection with the other.

Finite sets ok.

Countable sets.

Def a set is countable if it can be put in bijection with the counting numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$

Countable

Egs: the even numbers (on board)

the rational numbers (on board)...

<u>Cantor</u> ~~Th~~ $[0,1]$ is uncountable.
i.e.
"the infinity in the continuum is (much) greater than the infinity of counting numbers.

<u>Pf</u> : Use binary representation + contradiction.

$0 < x \le 1$; $x = .10110 1110 \cdots$
an infinite sequence of 0's & 1's.
$= \frac{1}{2} + 0 + \frac{1}{2^3} + \frac{1}{2^4} + 0 \cdots$

Suppose we have such a bijection.

$$1 \longleftrightarrow \sigma(1) = .\sigma(1)_1 \, \sigma(1)_2 \, \sigma(1)_3 \cdots$$
$$2 \longleftrightarrow \sigma(2) = .\sigma(2)_1 \, \sigma(2)_2 \, \sigma(2)_3 \cdots$$
$$\vdots$$
$$n \longleftrightarrow \sigma(n) = .\sigma(n)_1 \, \sigma(n)_2 \cdots$$

I am going to show we missed a number. Define a new sequence as follows:

1st $\bar{1} = 0$, $\bar{0} = 1$

$x = x_1 x_2 \ldots x_n \ldots$

whatever $\sigma(1)_1$ is x_1 is its opposite
" $\sigma(2)_2$ " x_2 is its opposite
etc.

ie.

$$x_n = \overline{\sigma(n)_n}$$

Claim: x is not on our list.

pf?
If it were,

$x = \sigma(k)$ some k,
so $x_k = \sigma(k)_k$ but
$x_k = \overline{\sigma(k)_k}$.

Woo~!