Feb 26 L1

Evaluating expressions which are formally 0/0

1) Recognize them as derivatives

Eg: $\frac{\sin x}{x} = \frac{\sin x - \sin 0}{x - 0}.$

& $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{x \to 0} \frac{f(x)}{x}$

if $f(0) = 0$ (like $\sin(x)$).

Simple expressions:

$\frac{3x}{x} \longrightarrow 3.$

$\frac{3x + 5x^2}{x} = 3 + 5x \longrightarrow 3.$

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so! $\dfrac{\sin x}{x} \longrightarrow \dfrac{d}{dx} \sin x \Big|_{x=0} = 1.$

since $\dfrac{d}{dx} \sin x = \cos x$ & $\cos 0 = 1$

How I think about it:

$\sin x \simeq x$

[More precisely:

$$\sin x = x - \frac{x^3}{3!} + O(x^5)$$

so

$$\frac{\sin x}{x} = \frac{x - \frac{x^3}{3!} + O(x^5)}{x}$$

$$= 1 - \frac{x^2}{3!} + O(x^4)$$

$\longrightarrow 1$ as $x \to 0$.

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L'Hopital

Suppose we know:

$f(0) = 0$, $g(0)$

& $f'(0)$, $g'(0)$ exist & are not both zero. Then:

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{f'(0)}{g'(0)}.$$

Proof 1: $\frac{f(x)}{x} \to f'(0)$

, $\frac{g(x)}{x} \longrightarrow g'(0)$ as $x \to 0$.

so $\frac{f(x)}{g(x)} = \frac{f(x)}{x} \cdot \frac{x}{g(x)} \to \frac{f'(0)}{g'(0)}$

Proof 2: $f(x) \simeq Kx + O(x^2)$

$g(x) \simeq lx + O(x^2)$

So $\frac{f(x)}{g(x)} \simeq \frac{Kx + O(x^2)}{lx + O(x^2)} = \frac{K}{l} + O(x)$

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& $k = f'(0)$,

$l = g'(0)$.

So $\dfrac{f(x)}{g(x)} \longrightarrow \dfrac{f'(0)}{g'(0)}$.

On Wed:

What to do if

$f'(0) = 0$

& $g'(0) = 0$?

Eg: $f(x) = 1 - \cos x$

$g(x) = x^2$.
