A 1

derived:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

"Newton's method for finding zeros".

In case of looking for roots:

$$x = \sqrt{N}$$

eg $\sqrt{3}$, $\sqrt{5}$.

$$f(x) = x^2 - N.$$

$$f'(x) = 2x$$

$$\frac{f(x)}{f'(x)} = \frac{x^2 - N}{2x} = \frac{1}{2}x - \frac{N}{2x}.$$

So:

$$x_{i+1} = x_i - \frac{1}{2}x_i + \frac{N}{2x_i}$$

$$= \frac{1}{2}\left(x_i + \frac{N}{x_i}\right).$$

did for $N=3$ last class

converges super-exponentially:

if $x_* =$ exact solution

$$|x_* - x_n| < C(\varepsilon_0)^{2^n}$$

Not proved.
maybe when do 2nd order Taylor series will prove.

¿Why does it work?

if

$$\frac{1}{2}\left(x_i + \frac{N}{x_i}\right) \longrightarrow x_*$$

then

$$\frac{1}{2}\left(x_* + \frac{N}{x_*}\right) = x_*$$

$\boxed{\text{Fixed Point}}$

so $\frac{1}{2}\frac{N}{x_*} = x_* - \frac{1}{2}x_* = \frac{1}{2}x_*$

$$\frac{N}{x_*} = x_* \Rightarrow N = (x_*)^2.$$

A 3

Possible Final Q

Work out the Newton iteration scheme for

$$x^5 = 17.$$