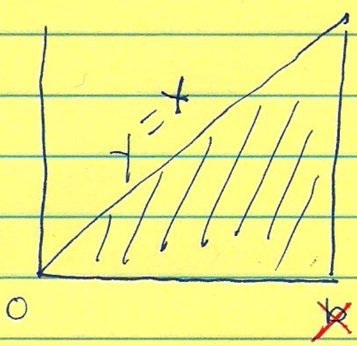
on to integration.

Q.
1 What is the area of the shaded region?:



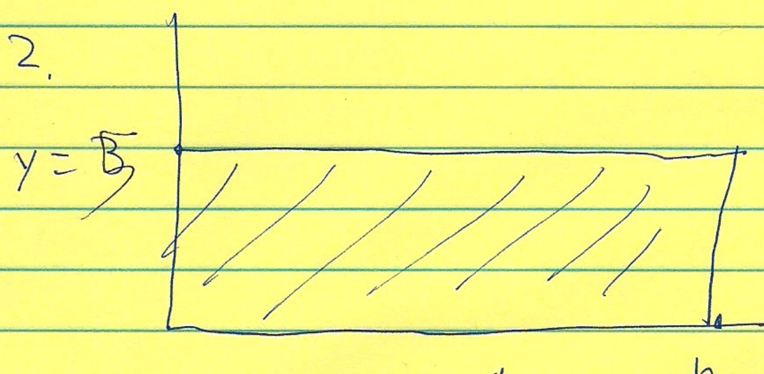
quiz results

? $\frac{1}{2}x^2$

Symbol for area: $\int_0^x s\,ds$. or $\int_0^x t\,dt$.

or: Abuse of notation. $\int_0^x x\,dx$

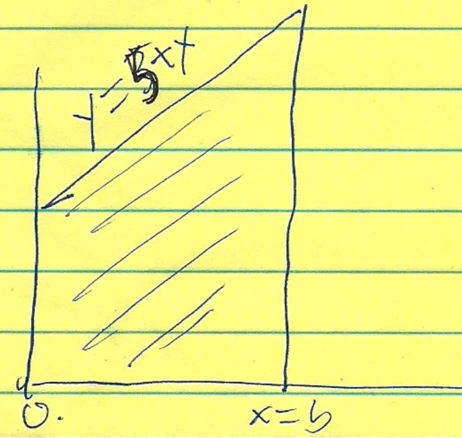
2.



?

$5x$

3.



$5x+\frac{1}{2}x^2$.

Observe:

$$\frac{d}{dx}\left(\frac{1}{2}x^2\right) = x$$

$$\frac{d}{dx}(5x) = 5$$

$$\frac{d}{dx}\left(5x + \frac{1}{2}x^2\right) = 5 + x$$

— General result: —

Write: "area under curve $y = f(x)$" from 0 up to variable point x:

as

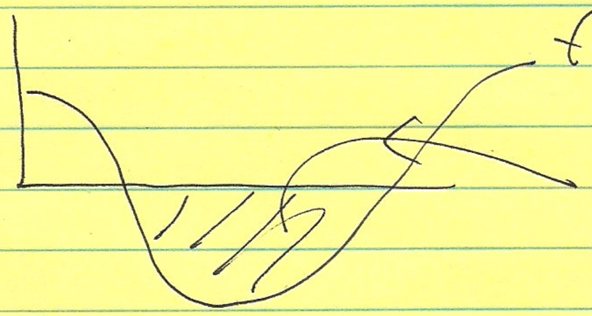
$$\int_0^x f(s)\,ds := F(x)$$

use dummy variable so as to not confuse with endpoint x.

Then: F is an antiderivative of f:

$$\frac{dF}{dx}(x) = f(x).$$

Take care at negative values



Integral sign is for sum.

Eg:

$\int_0^1 x^5 dx$ = Sum the function x^5 over all x's from $x = 0$ to 1.

$$= \int_{x=0}^{1} x^5 dx$$

needed for many reasons; the 'increment' to sum over.

Why am I doing this now?
Doesn't it belong in 19B

Why are you teaching us to add? Shouldn't we spend the first quarter subtracting?

or:

"First we will teach you to walk just with your right leg. After you have mastered that we will allow you to use your left leg."

Differentiation — Integrat

f'

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

≡ Pause ≡

Point out:

youtube.

HWs.

quiz next week

Qs?

math?

admin?

quiz & HW scores
toss out.

In calculus functions come in pairs.

$$f \longrightarrow F = \int^x f$$

$$f = F' \longleftarrow F$$

Most of the game is the interplay between the two... operations & :

$$\text{"} \int^x \frac{d}{dx} = \frac{d}{dx} \int^x \text{"} = \text{Identity.}$$

Definition of "Area under graph"

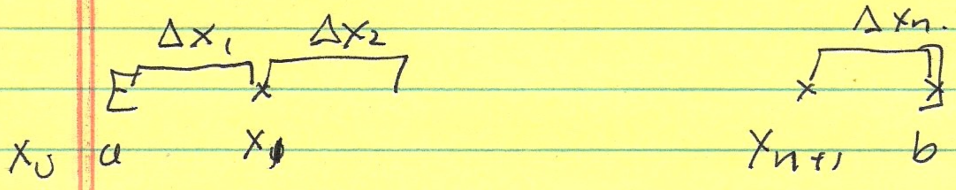
$$\int_{x=a}^{x=b} f(x)\,dx$$

See Strang's lecture!
MIT

Theory & Numerics

Partion $[a, b]$ into a finite number of segements:

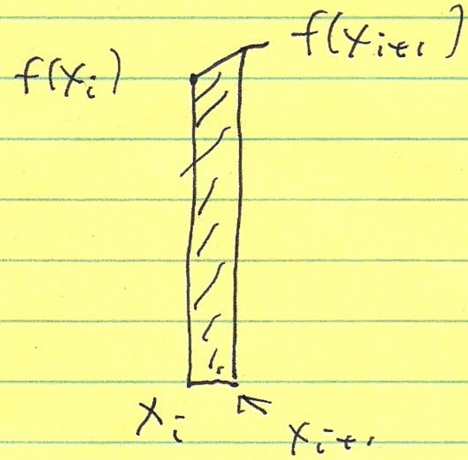
$$a = x_0 < x_1 < x_2 < \cdots < x_n = b.$$



Replace, over each subinterval $[x_i, x_{i+1}]$

the alleged $\displaystyle\int_{x_i}^{x_{i+1}} f(x)\,dx$

By some approximate guess: A_i



$$\Delta x_i = x_{i+1} - x_i$$

Rectangles.

$$A_i = (\Delta x_i) f(x_i) \quad \text{or} \quad (\Delta x_i) f(x_{i+1})$$

or $\cancel{\Delta x}\ \Delta x_i \frac{1}{2}\left(f(x_i) + f(x_{i+1})\right)$ or - -

← Trapezoid.

Then since

$$\int_a^b f(x)dx = \int_{x=a}^{x=x_1} f(x)dx + \int_{x=x_1}^{x=x_2} f(x)dx + \cdots$$

$$\& \int_{x_i}^{x_{i+1}} f(x)dx \simeq A_i$$

Summation Notation!

Get

$$\int_a^b f(x)dx \cong \sum_{i=1}^{n} A_i$$

Theorem if f is continuous or only piecewise continuous then as $\Delta x_i \to 0$, so also $N \to \infty$

$$\sum_{i=1}^{N} A_i \longrightarrow \text{limit}.$$

$$\text{limit} = \text{Integral}.$$

$$= \int_a^b f(x)dx.$$

Theory & Numerics

If you use "first guess"

$$f(x_i)\Delta x_i = A_i$$

or any $f(x_i^*)\Delta x_i$

for $x_i \le x_i^* \le x_{i+1}$

& if $\Delta x_i = \dfrac{a-b}{n}$

(equal spacing)

This is the standard Riemann sum approximation.

$\boxed{\text{Any method yields the same answer.}}$

Riemann

$$\int_a^b f(x)\,dx = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \sum_{i=1}^{N} f(x_i)\Delta x_i$$

eg: $x_i = a + \dfrac{i(b-a)}{N}$ $\quad i = 0, 1, \ldots, N.$

$$\Delta x_i = \frac{b-a}{N}.$$

In practice use

Fundamental theorem of calculus:

1): $\int_a^b f'(x)\,dx = f(b) - f(a)$,

Also written:

$$\int_a^b f(x)\,dx) = F(b) - F(a)$$

where F any function such that $F'(x) = f(x)$.

Eg $\int_0^1 x\,dx$.

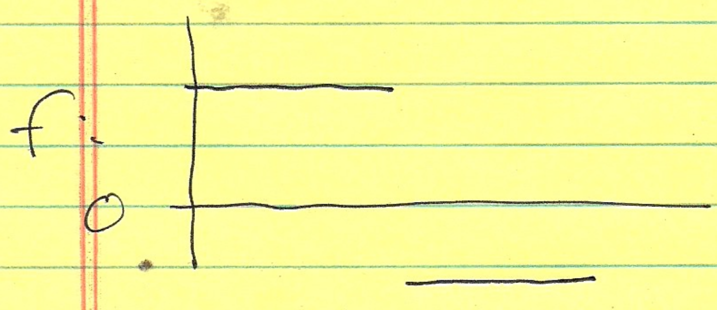
$f(x) = x = F'(x)$

if $F = \frac{1}{2}x^2$.

$\int_0^1 x\,dx = F(1) - F(0)$

$= \frac{1}{2} 1^2 = \frac{1}{2}$.

Time permitting.



$\boxed{F = ?}$