1.

Now x's in texts.

$\sqrt{}$

$f(x)$ $F(x)$ or $\int f(x)dx$.

function $\longleftrightarrow$ an antiderivative;

$f = F'$ F only defined up to a constant

Have someone read Fund thm from clip out of "Single Variable.."

~~Do~~: Note $\int_0^x f(t)dt = F(x)$

has property that $F(0) = 0$

While $\int_1^x f(t)dt = G(x)$

has property that $G(1) = 0$.

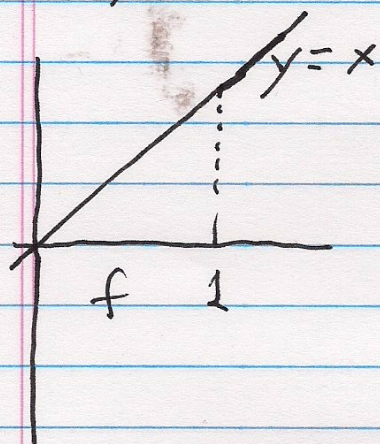
$G(x) = F(x) + C$

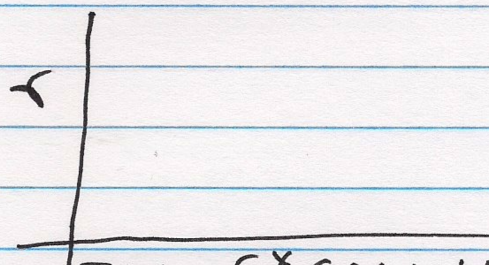
Both antiderivatives of f.

---

in class.

Exer:

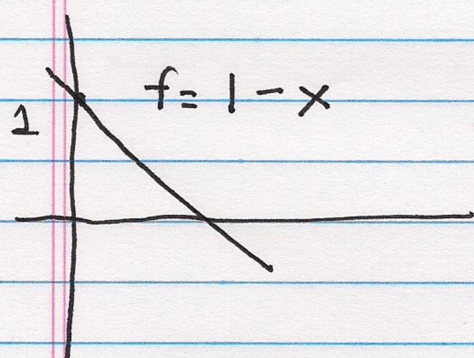


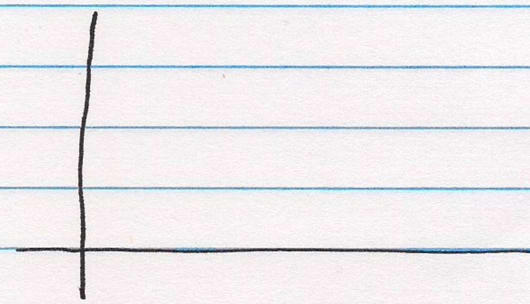


$F(x) = \int_0^x f(t)\,dt$

call F_+

vs.





$F = \int_0^x f(t)\,dt.$

call F_-

Note: for both: ~~...~~ $F(1) = \frac{1}{2}$.

why? From picture?

Call them F_+ & F_-

$F_+(1) = \frac{1}{2} = F_-(1)$ but.

$F_+(0) = 0 = F_-(0)$

$F_-'(\frac{1}{2}) = 0.$

Why? ...

picture & algebra

& area.

---

what is $F_+'(\frac{1}{2}) = ?$

Reading ~~speed~~ velocity argument

for validity of F.T.C.

from clip. of "Single Variable.."

Why is F.T.C. true? II

Discrete model: Example:

F: 1 2 4 6 7 9 8 6

f: 1 2 2 1 2 −1 −2.

How did I get f values from F?

$$1 + 2 + 2 + 1 + 2 - 1 - 1 = 6 - 1$$

why?

$$(2-1) + (4-2) + (6-4) + (7-6) + (9-7) + (8-9) + (6-8)$$

$$= (6-8) + (8-9) + (9-7) + (7-6) + (6-4) + (4-2) + (2-1)$$

<u>telescoping sum</u> !

$$\underline{= 6 - 1}.$$

Symbols:

$$f(x_i) = F(x_{i+1}) - F(x_i).$$

eight values of F

seven of f :

$x_0 \leftrightarrow a$, $\quad x_7 \leftrightarrow b$.

$$\int_a^b f(x)\,dx = \sum_{i=0}^{6} f(x_i)$$

$$= \sum_{i=0}^{6} \left(F(x_{i+1}) - F(x_i)\right)$$

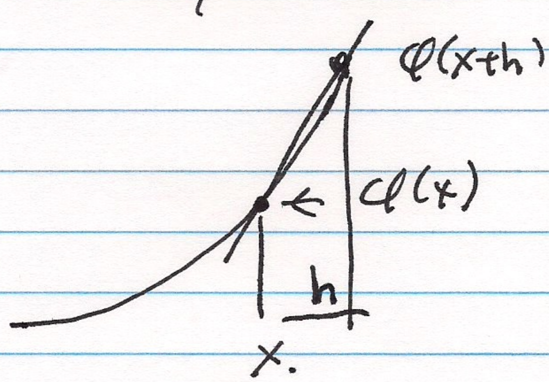
$$= \text{telescope!}$$

$$F(x_7) - F(x_0)$$

$$= 6 - 1.$$

Using limits: ...

Any function φ.



$$\varphi'(x) = \lim_{h \to 0} \frac{\varphi(x+h) - \varphi(x)}{h}$$

"secants limit to the tangent"

Us:

we will use:

$$F(x) = \int_a^x f(t)\,dt.$$

& $$\int_a^b f(t)\,dt + \int_b^c f(t)\,dt$$

$$= \int_a^c f(t)\,dt.$$

$$\left(\text{like} \quad \sum_{i=1}^{20} f_i = \sum_{i=1}^{13} f_i + \sum_{i=14}^{20} f_i \right)$$

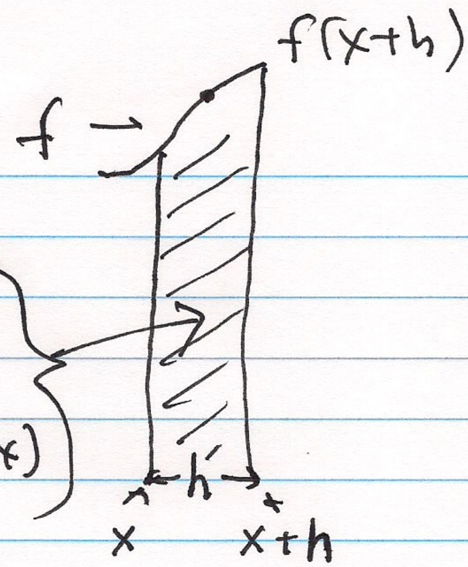
so:

$$F(x+h) = \int_a^{x+h} f(t)\,dt = \int_a^x f(t)\,dt + \int_x^{x+h} f(t)\,dt.$$

or

$$F(x+h) - F(x) = \int_x^{x+h} f(t)\,dt.$$

Now look at meaning:



ΔF
$=$
$F(x+h)-F(x)$

$$F(x) = \int_a^x f(t)\,dt$$

ct'd

area is $\simeq h \cdot f(x)$

$\simeq h \cdot f(x_*)$

using:
f continuous. → any x_*, $x < x_* < x+h$

so

$$\frac{F(x+h)-F(x)}{h} \simeq f(x_*)$$

so

& as $h \to 0$ $\quad F'(x) = f(x)$

the

$\simeq$ becomes

an

"$=$"

LIMITS!