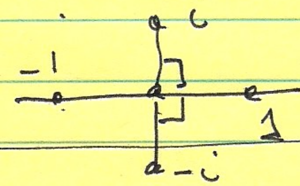
Quiz 1) $Ax^2 + Bx + C = 0$
with $A, B, C$ constants
what is $x$ ?

2) $x^2 + 1 = 0$.
what is $x$ ?




Recall $P(t)$ solving

$$\frac{dP}{dt} = KP \qquad K \text{ constant.}$$

Then $P(t) = Ae^{Kt}$ $\quad A$ constant.

eg. $K = \log_e 2$; $\quad e^{Kt} = (e^K)^t = 2^t$

then $P(t) = A \cdot 2^t$. ; $P(0) = A$

What if ~~P~~ $K = i = \sqrt{-1}$ ?

$$\frac{dP}{dt} = i\,P \quad ; \quad P = Ae^{it} = ???$$

real, imag. real.

Need $P(t) = f(t) + i g(t)$.

then $\dfrac{dP}{dt} = \dfrac{df}{dt} + i \dfrac{dg}{dt}$

$$i(f+ig) = if - g$$

$$= -g + if.$$

$$\frac{df}{dt} + i\frac{dg}{dt} = -g + if.$$

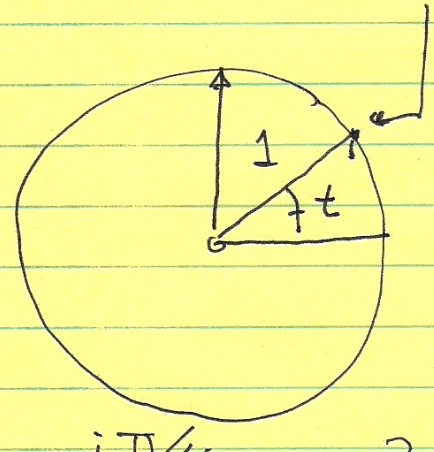
so $$\frac{df}{dt} = -g$$

$$\frac{dg}{dt} = f$$

?? $f = \cos(t)$, $g = \sin(t)$ works.

<u>Euler</u>!

$$e^{it} = \cos t + i \sin t$$



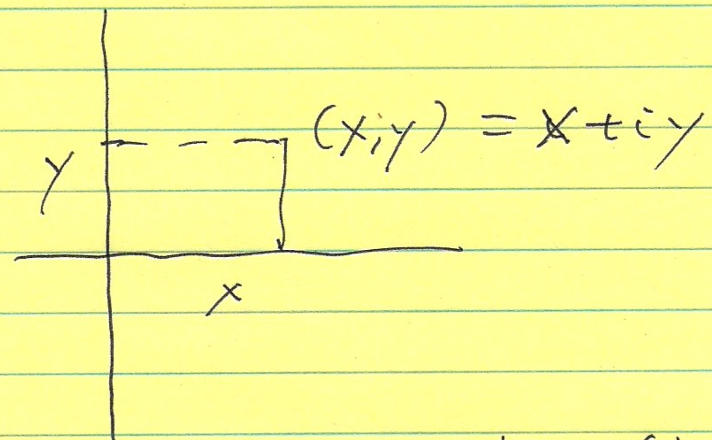

so: $e^{i\pi/2} = i$

$e^{i\pi} = -1$

$e^{i3\pi/2} = -i$

$e^{i2\pi} = i$

$$e^{i\pi/4} = ? + i ?$$




$\mathbb{R}^2 \simeq \mathbb{C}$ so $1 = (1,0)$
$i = (0,1)$.

All trig identities are contained in Euler's formula.

Eg. Angle addition:

$$e^{i(\theta_1+\theta_2)} = e^{i\theta_1} e^{i\theta_2}$$

do double $\measuredangle$ instead.

$$\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2) = (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$

Work it out. $\to$

linearizations
Taylor series

If $y'(x) = y$
& $y(0) = 1$
Guess: & $y = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \cdot$
then
$y = y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3$

$$\Rightarrow \begin{aligned} 1 &= a_1 \\ a_1 &= 2a_2 \\ a_2 &= 3a_3 \\ a_3 &= 4a_4 \\ &\vdots \end{aligned} \quad \Rightarrow \quad \begin{aligned} a_1 &= 1 \\ a_2 &= \tfrac{1}{2} \\ a_3 &= \tfrac{1}{3} a_3 = \tfrac{1}{3!} \\ a_4 &= \tfrac{1}{4} a_3 = \tfrac{1}{4!} \end{aligned}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \cdots$$

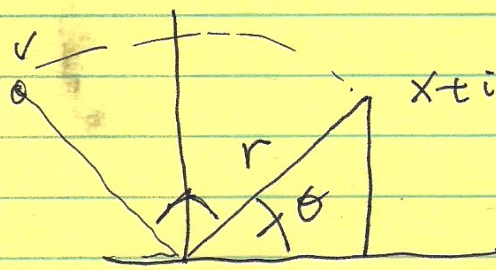
$$= \left(1 - \frac{x^2}{2} + \cdots\right) + i\left(x - \frac{x^3}{3!} + \cdot\right)$$

$$= \cos x + i \sin x.$$

$$\rightarrow \left.\begin{aligned} \cos x &= 1 - \frac{x^2}{2} + \cdot \\ \sin x &= x - \frac{x^3}{3!} + \cdots \end{aligned}\right\} \text{ very useful!}$$

geom meaning etc

$re^{i(\theta+\pi/2)}$




$x+iy = re^{i\theta}$

$r = \sqrt{x^2+y^2}$

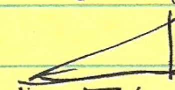
$r\cos\theta = x$

$r\sin\theta = y.$

Rotate by $90°$ = Multiply by $i = e^{i\pi/2}.$

Rotate by $30°$ ?

Multiply by $e^{i\pi/6}$

= 


$= \frac{\sqrt{3}}{2} + \frac{i}{2}.$

etc etc.

Spirals. $\frac{dP}{dt} = (k_1+ik_2)P(t)$

$P(0)=1$

Soln $P(t) = e^{(k_1+ik_2)t}$

$= e^{k_1t}e^{ik_2t}$

$= e^{k_1t}(\cos k_2t + i\sin k_2t).$

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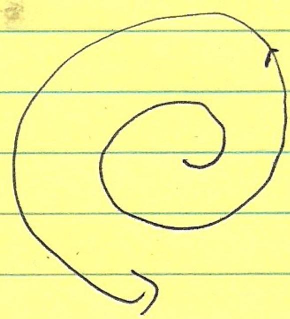
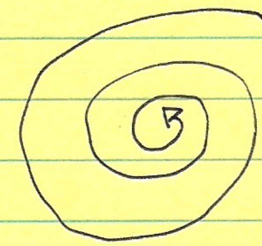
$K_1 > 0$, $K_2 > 0$ ; $K_1 < 0$, $K_2 > 0$





$r(t) = e^{K_1 t}$

$\theta(t) = K_2 t$.

$Re(P(t)) = e^{K_1 t} \cos K_2 t$

$Im(P(t)) = e^{K_2 t} \sin K_2 t$.

graphs ? - - .

(Midterm !).