Rules of exponents.

$$(b^4)^5 = b^{4\cdot 5} \; ; \; b^4 \cdot b^5 = b^{4+5}$$

Example:

$$(2^2)^3 = (2\cdot 2)^3 = \underbrace{(2\cdot 2)(2\cdot 2)(2\cdot 2)}_{6 \;\; 2\text{'s} \; !}$$

$$= 2^6 \; , \; (2^2)^3 = 2^{2\cdot 3}$$

Example

$$2^2 \cdot 2^3 = \underbrace{(2\cdot 2)(2\cdot 2\cdot 2)}_{5 \;\; 2\text{'s}}$$

or

$$2^2 \cdot 2^3 = 2^{2+3} = 2^5$$

$1\frac{1}{2}$ $\emptyset$

exponentiating.
(any base).

Turns adding to multiplying.

$$\exp(x) = e^x \quad (\text{or } b^x,\ b > 0).$$

$$\exp(x+y) = \exp(x)\exp(y)$$

$$\Rightarrow \begin{cases} \exp(x)^2 = \exp(2x) \\[6pt] \dfrac{1}{\exp(x)} = \exp(-x) \\[6pt] \exp(0) = 1 \end{cases}$$

$$(\exp(x))^n = \exp(nx)$$

Why? How ...?

$$\frac{d}{dx} e^{Kx} = \left(e^{Kx}\right) \frac{d}{dx}(Kx)$$

$$= \left(e^{Kx}\right) K.$$

$$= K e^{Kx}.$$

K a constant ; eg an integer.

But $e^{K} = b =$ some other constant.

so $e^{Kx} = (e^{K})^{x} = b^{x}$

Thus

$$\frac{d}{dx} b^{x} = K \cdot b^{x} = K e^{Kx}$$

where $e^{K} = b$

i.e. $K = \log_e(b)$

~~(9)~~

Case 1 $\boxed{B}$

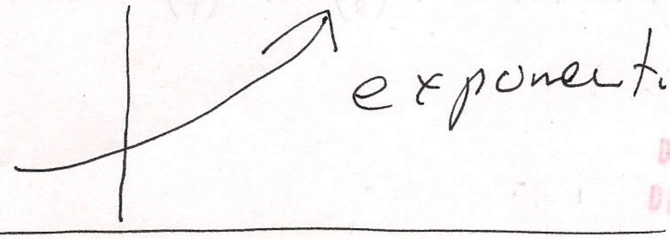
$1 < b < \infty$.

Example: $b = 2$.

as $x \nearrow$ $b^x \nearrow$.

$2, 2^2, 2^3 = 8, 2^4 = 16$

$b^x = e^{kx}$ with $b > 1$ so $k > 0$



exponential growth.

Case 2. $0 < b < 1$

so $k < 0$, Eg $\frac{1}{e} = b$, $0 < \frac{1}{e} < 1$

$e^{-1} = \frac{1}{e}$; $k = -1$.

$e^{kt} = b^t$

as $t \nearrow$, $e^{kt} = b^t \searrow$

eg: $\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3 \ldots$

$1 > a > 0$

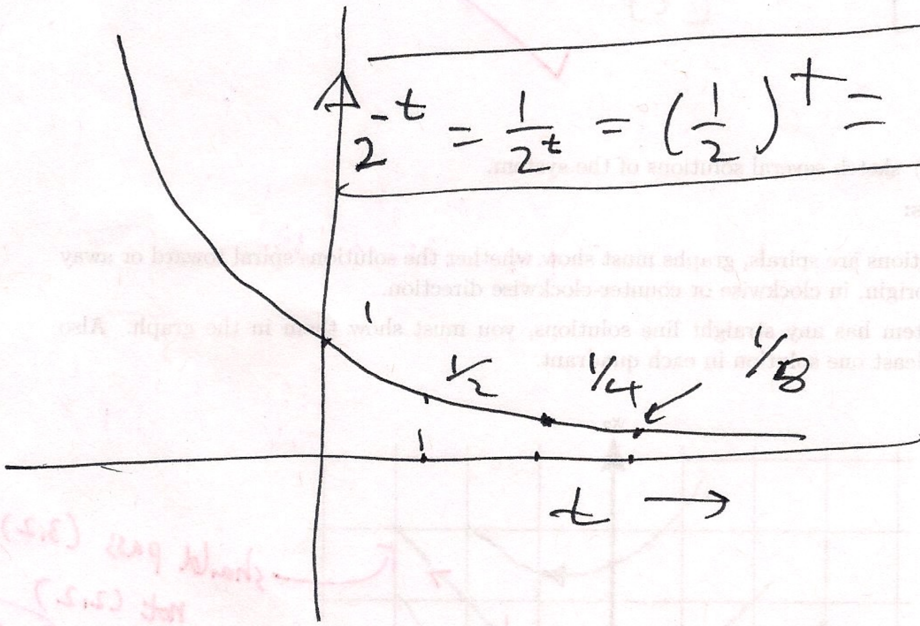
case

eg $\left(\frac{1}{2}\right)^t = \frac{1}{2^t} = e^{Kt}$ 4

$K = -\log_e 2.$

$= +\log_e\left(\frac{1}{2}\right).$

$$\boxed{2^{-t} = \frac{1}{2^t} = \left(\frac{1}{2}\right)^t = e^{-(\log_e 2)t}}.$$



Exponential Decay