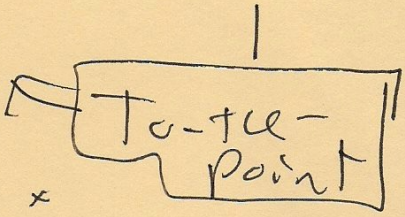
(1) Re-review exponentials.




(2) *exp: turns + into ×

$\exp(x+y) = \exp(x)\exp(y)$

*log turns × into +.

Algebraic "additive identity": 0

"goes to multiplicative" identity.

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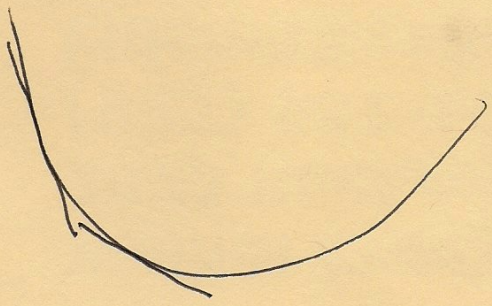
(2) Review rules of differentiation (thanks to Reshee!)

(3) $f''$ & convexity/concavity

Ruler



convex

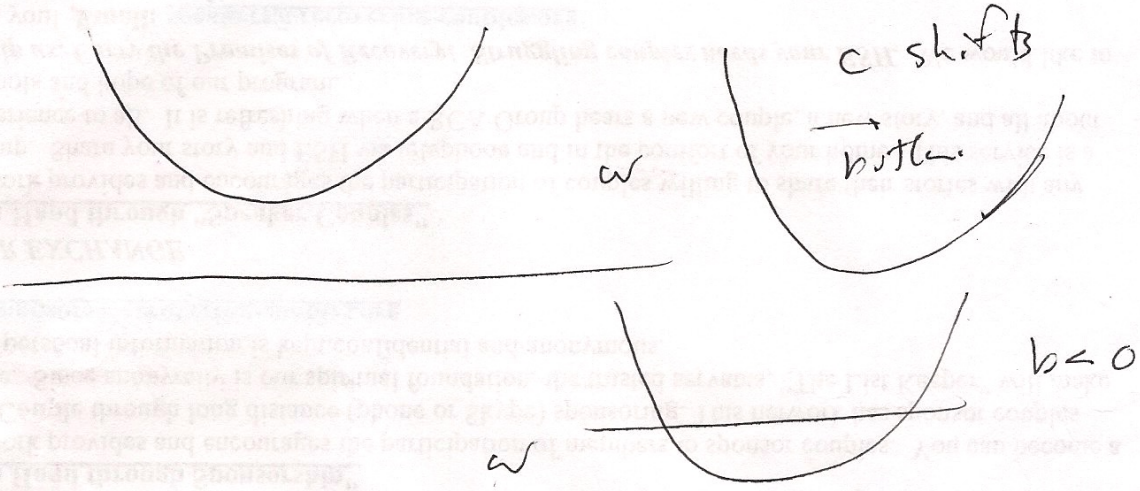
→ Slope steadily increases as more along curve.

$(f')' > 0$. $f'' \geq 0$

Eg $f'' = 1$

so $f' = x + c$.

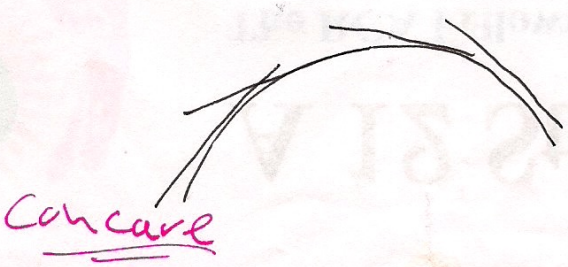
$f = \frac{1}{2}x^2 + cx + b$.

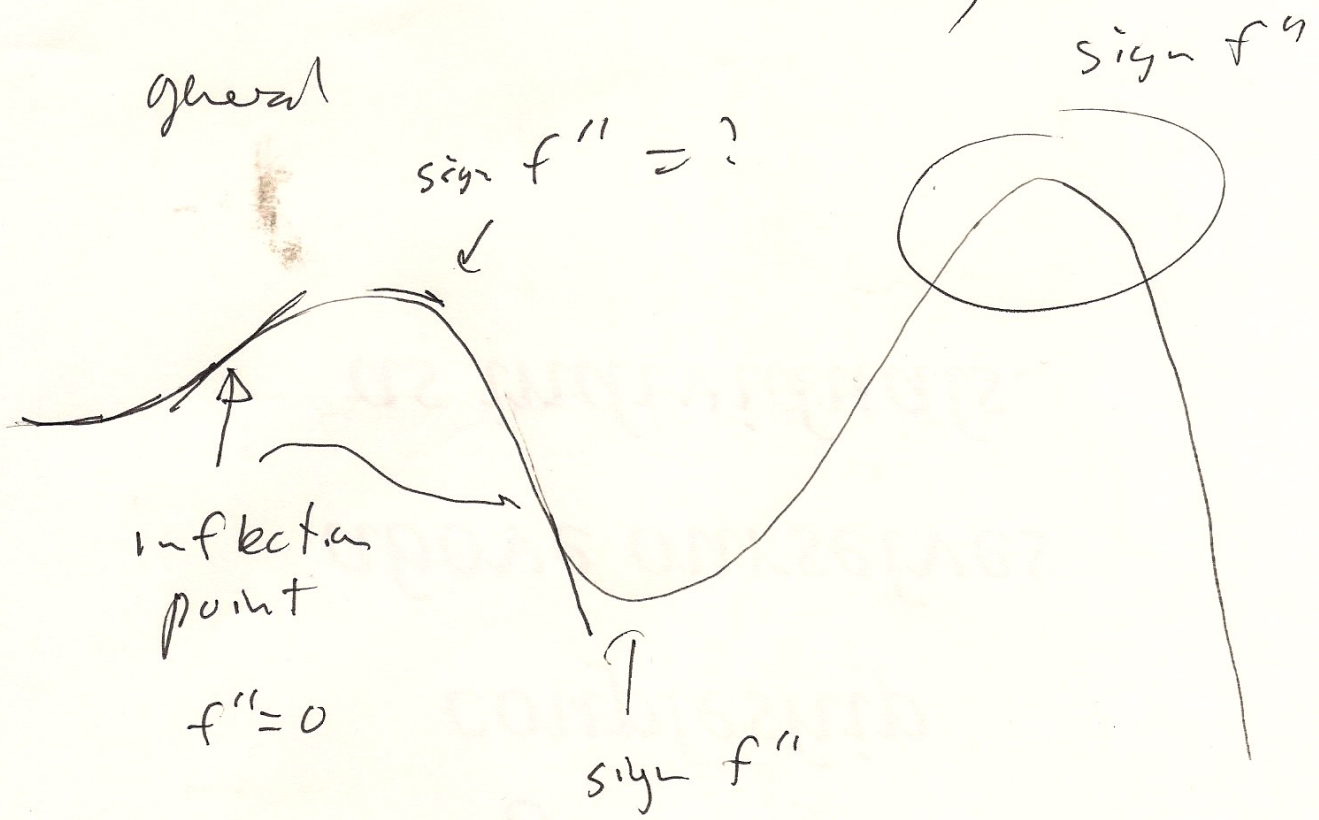



eg. $\frac{d^2f}{dt^2} = -k$, c a constant

eg $k = 16 \frac{m}{s^2}$.

$f = -\frac{1}{2}kt^2 + bt + c$.



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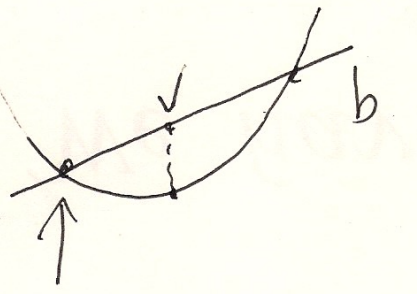
def of "convex"

$$f((1-t)a+tb) \leq (1-t)f(a) + tf(b)$$




$0 \leq t \leq 1$

$t=0$ a; $t=1$ b

Then in analysis,

$$f\left(\tfrac{1}{2}a + \tfrac{1}{2}b\right) \leq \tfrac{1}{2}f(a) + \tfrac{1}{2}f(b)$$

is enough.

~~but~~ If equality flips
$\cup$ concave.

Exer

C: Which is convex, which is concave?

$e^x$

or $\log x$

(hint: graph 'em!)