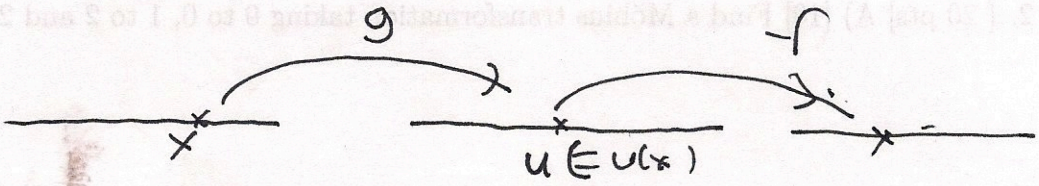

$$f(g(x)) = f(u) = \quad \underset{u=g(x).}{\uparrow}$$

$$\frac{d}{dx} f(g(x)) = \frac{df}{du}\frac{du}{dx}$$

$$\underset{u=g(x)}{\uparrow}$$

or

$$f(g(x))' = f'(g(x))\,g'(x).$$

In case f, g both linear in x. f' & g' are constants

$$\text{If } f(x) = mx + b$$
$$g(x) = lx + a.$$

$$f(g(x)) = m(lx+a)+b$$
$$= mlx + \text{const}.$$

$$f(g(x))' = ml = f'g'$$

But if they are nonlinear, take some care:

Eg $\frac{d}{dx}\sqrt{x^2+1}$.

$f(u) = \sqrt{u}$.

$g(x) = x^2+1 \qquad (= u(x))$

$$\frac{df}{du} = \frac{1}{2} u^{-1/2}.$$

$$\frac{dg}{dx} = 2x.$$

$$\frac{df}{du}\frac{du}{dx} = \frac{1}{2} u^{-1/2} \cdot 2x.$$

$$= \frac{1}{2} \frac{1}{\sqrt{x^2+1}} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+1}}.$$

You do:

$$\frac{d}{dt} e^{(.1)t}$$

$$(\sin 3x)'$$

$$\frac{d}{dx}\left(e^{-x^2}\right)$$

Suggestion: Active Calculus

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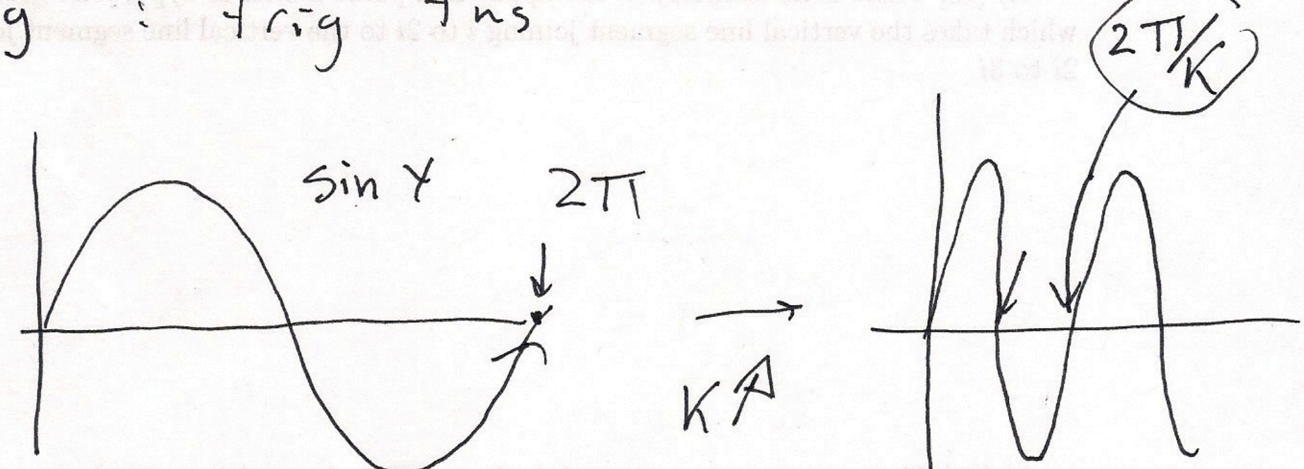
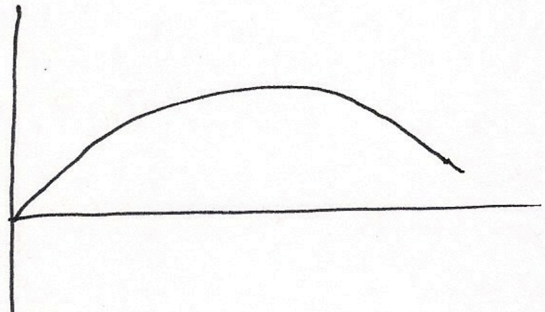
To "speed up" or "slow down" a function $f(x)$

Replace with $f(Kx)$, K const.

Eg: trig fns



K ↓

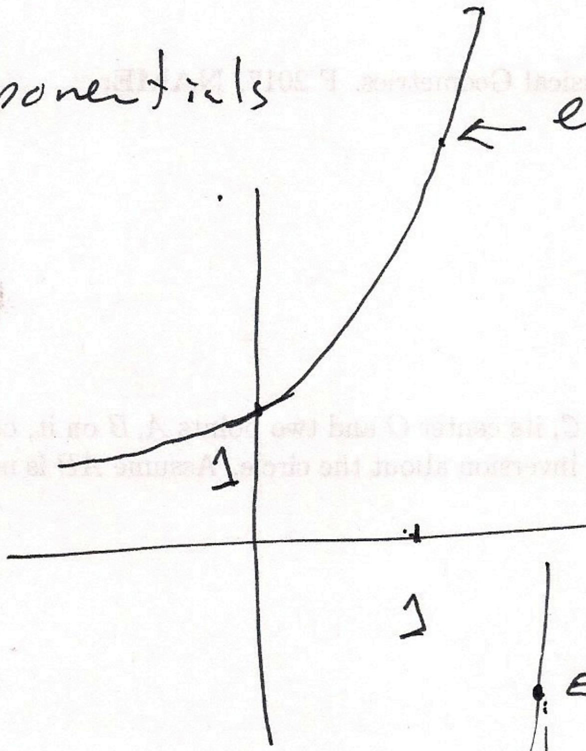


Suggestn ~~Alt~~ Wolfram Alpha or Mathematica.

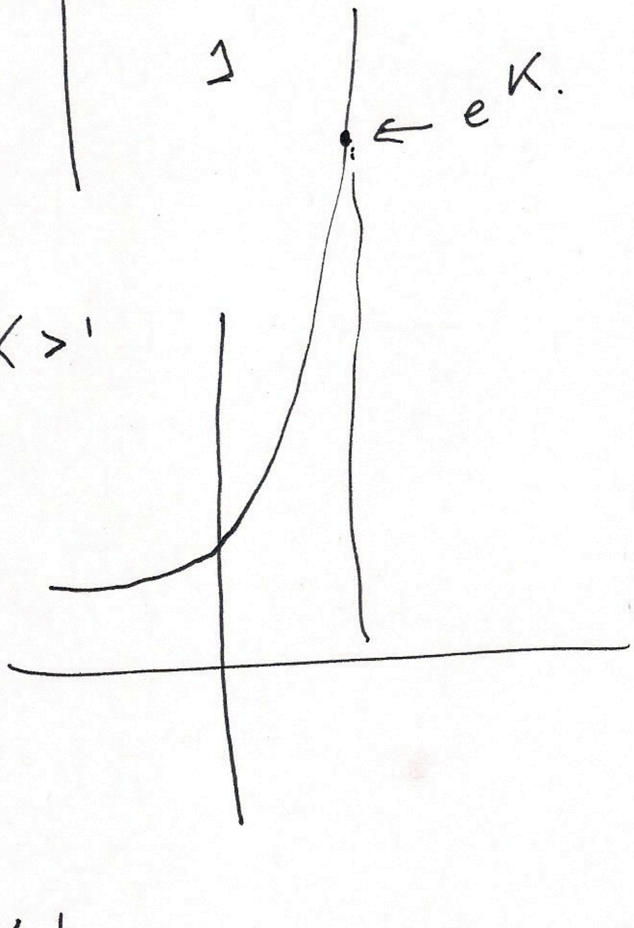
5

Exponentials

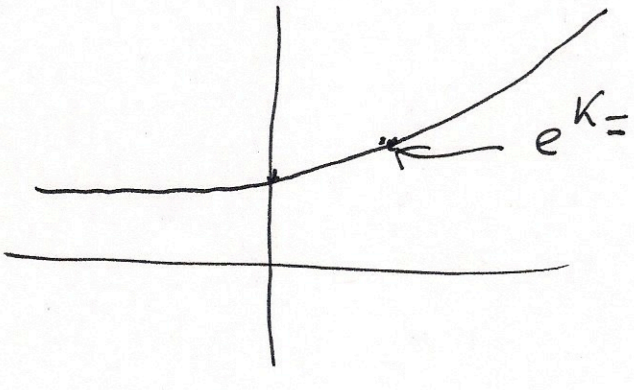
e^t



e^{Kt} ; $K > 1$

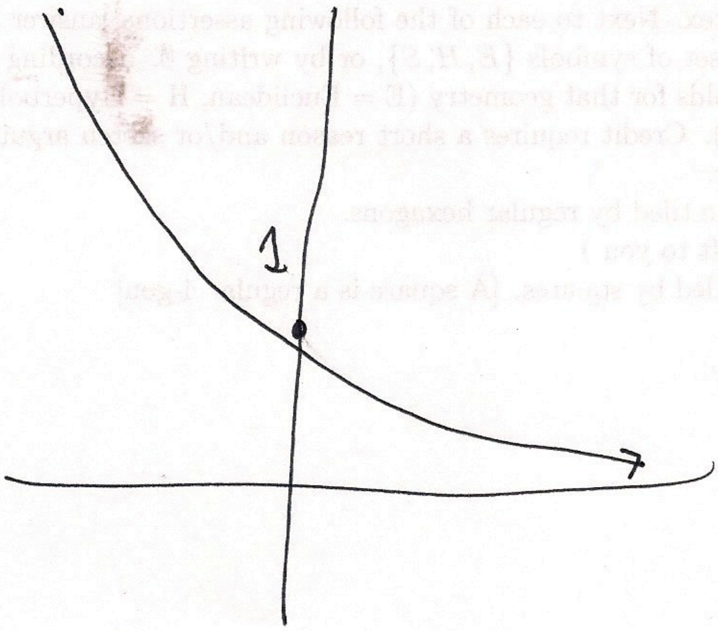


$0 < K < 1$



6

e^{-Kt} $\quad K > 0$



$$\frac{d}{dt} P(t) = K P(t).$$

$$\iff P(t) = Ae^{Kt}.$$

Meaning of A?

of K?

Using chain rule to compute derivatives of inverse functions.

Eg. $f(x) = x^2$.

$$f^{-1}(x) \underset{?}{=} \sqrt{x}, \quad x \geq 0$$

$$\sqrt{x^2} = x, \quad (\sqrt{x})^2 = x,$$

Write $g = f^{-1}$

$$f(g(x)) = x \; ; \quad f(u) = u^2$$

$$\frac{d}{dx} \boxed{f'(g(x)) \cdot g'(x) = 1}$$

$$f'(u) = 2u.$$

$$2g(x) \cdot g'(x) = 1 \Rightarrow g' = \frac{1}{2g} = \frac{1}{2\sqrt{x}}.$$

Trig function inverses

Now -

$$\frac{d}{dx}\sin^{-1}(x) \overset{?}{=}$$

$$\frac{d}{dx}\tan^{-1}(x) \overset{?}{=}$$

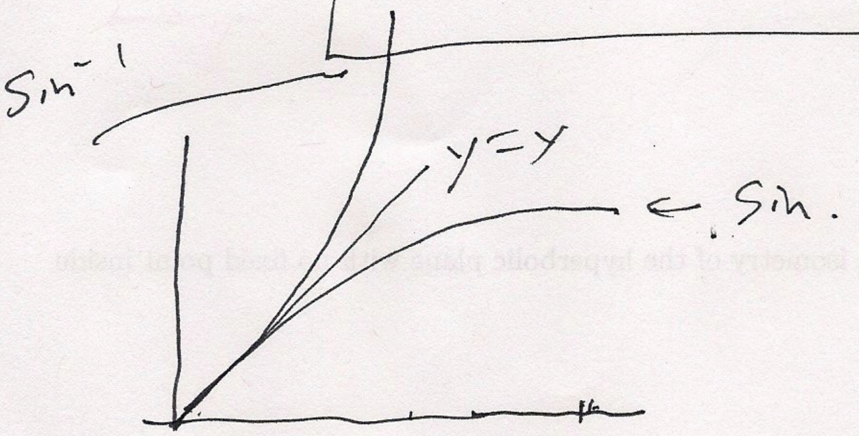
← **Needed for Mon HW.**

terminology.

$\sin^{-1}$ = "Arcsin"

$\tan^{-1}$ = Arctan.

~~Well~~. ★



$\sin\frac{\pi}{2} = 1$

$\text{Arcsin } 1 = \frac{\pi}{2}$.

$$\boxed{\sin(\text{Arcsin } x) = x}$$

$$\boxed{\text{Arcsin}(\sin\theta) = \theta}$$

Same game:

$$\frac{d}{dx}\left[\left(\sin(\text{Arcsin}(x)\right) = x\right]$$

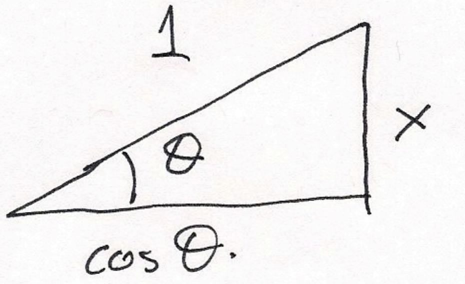
$$\sin'(\theta)\frac{d}{dx}\text{Arcsin}\, x = 1.$$

$$\cos\theta \, \frac{d}{dx}\text{Arcsin}\, x = 1.$$

$$\theta = \text{Arcsin}\, x.$$

$$\Rightarrow \frac{d}{dx}\text{Arcsin}\, x = \frac{1}{\cos\theta} \stackrel{?}{=} ?(x)?$$

Pythageras



$$\Rightarrow \cos\theta = \sqrt{1-x^2}$$

since $x^2+\cos^2\theta = 1.$

$$\Rightarrow \boxed{\frac{d}{dx}\text{Arcsin}(x) = \frac{1}{\sqrt{1-x^2}}}$$

$y = y(x)$

Invert (suppose)

$x = x(y)$.

eg: $y = \sin x$; $x = \text{Arcsin}\, y$.

then

$$\frac{dx}{dy} = \frac{1}{\left.\frac{dy}{dx}\right|_{x = x(y)}}$$

$=$