Quiz 1/17. $\sqrt{10}$

$10^3 = 1{,}000.$

Use the linear approximation to estimate

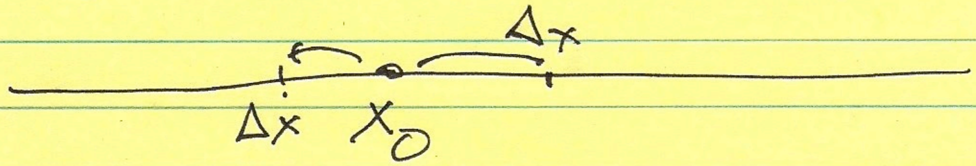
$$\sqrt[3]{1003}$$

Maybe redo for, say $\sqrt{97}$. or $\sqrt{80}$.

Linear approximation to function f near point x_0.

We imagine we know $f(x_0)$ & want to know it for nearby values

$$x = x_0 + \Delta x$$



$$f(x_0 + \Delta x) \simeq f(x_0) + f'(x_0)\Delta x$$

is the linear approximation.

Estimating $\sqrt{10}$ using the linear approx of Calculus.

$$10 = 9 + 1$$
$$= x_0 + \Delta x$$

So with $f(x) = \sqrt{x}$.

$$f(10) \simeq f(\underset{\underset{x_0}{\uparrow}}{9}) + f'(9) \cdot \underset{\underset{\Delta x}{\uparrow}}{1}$$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2} \frac{1}{\sqrt{x}}$$

(Power law)

So

$$f'(9) = \frac{1}{2} \frac{1}{\sqrt{9}} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}.$$

$$\Rightarrow \sqrt{10} = \sqrt{9} + \frac{1}{6} \cdot 1$$

$$\boxed{\sqrt{10} \simeq 3 + \frac{1}{6}.}$$

cf 3.166... w/ calculator.

Why does it work, the linear approximation?

$$f(x_0+\Delta x) \simeq f(x_0) + f'(x_0)\Delta x$$

$$\Longleftrightarrow \quad f(x_0+\Delta x) - f(x_0) \cong f'(x_0)\Delta x$$

$$\Longleftrightarrow \quad \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} \simeq f'(x_0)$$

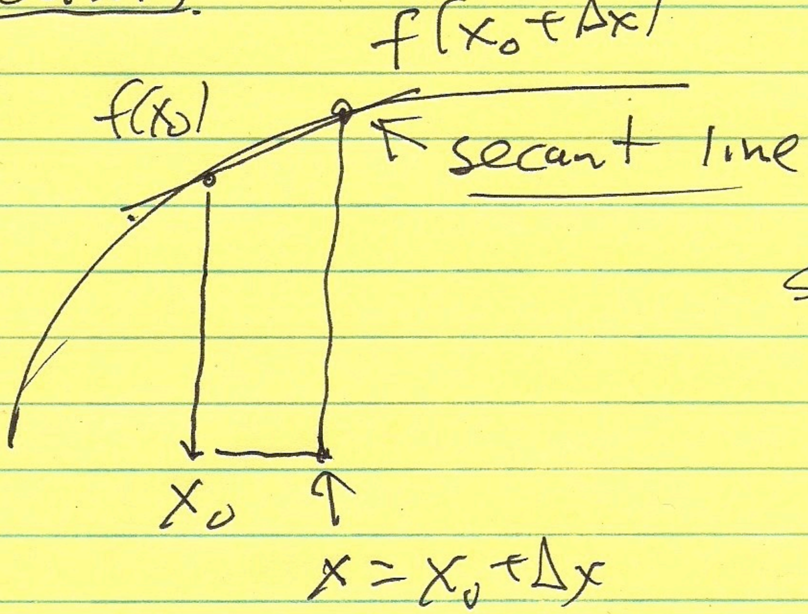
This is <u>almost</u> the analytic def of the derivative!

$$\boxed{\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = f'(x_0)}$$

Relation to 1st description of the derivative:

slope of tangent ~~of~~ to graph of $f(x_0)$ at the point $(x_0, f(x_0))$.

Secants



slope of secant

$$\frac{\text{rise}}{\text{run}} = \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

as $\Delta x \to 0$ secant converges to tangent . . .

so limit goes to $f'(x_0)$.

---

3 perspectives on derivative

L1 slope of tangent

L2 rate of change: if $f(t)$ is ~~true~~ elapsed dist. travelled up to time t, the

$$\frac{df}{dt}(t) = \text{instantaneous velocity}$$

&

L3: Best linear approx to $f(x)$ near x_0.

$\Delta x = x - x_0$.

$f(x_0) + f'(x_0)(x - x_0) = \ell(x)$

$\ell(x_0) = f(x_0)$

Slope of $\ell = f'(x_0)$.

practical!
used in numerics all the time.

My problems?:
do one more.

So far:

Can compute:

$\frac{d}{dx} P(x)$ a polyn.

$\frac{d}{dx} x^{\alpha} = \alpha x^{\alpha-1}$

eg: $\frac{d}{dx} x^{\frac{1}{2}}$.

$\frac{d}{dx}(e^{x}) = ?$

Next: $\frac{d}{dx}\cos(x)$, $\frac{d}{dx}\sin x$.

Then if ~~f, g~~ we know

$f'(x)$, $g'(x)$ ~~how to~~

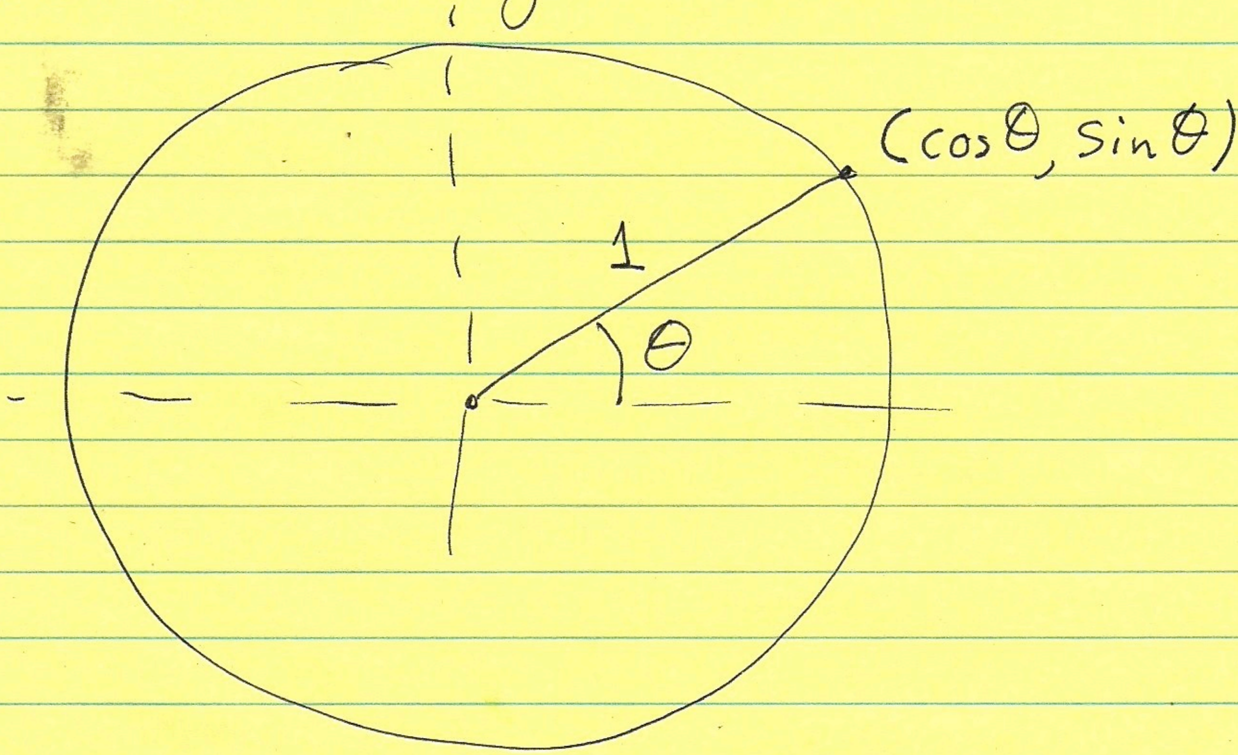
how to compute $(f(g(x)))'$

---

eg: $(e^{-x^2})' = ?$

$f = e^{u}$; ~~g(x)~~ $u = -x^2$.

Review trig.



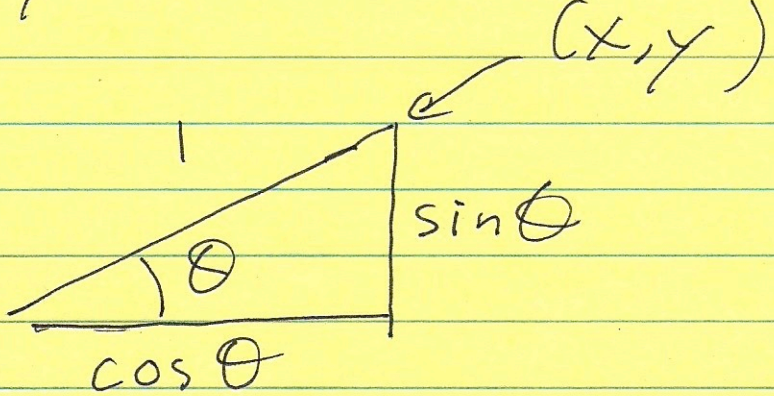
As θ varies

$x = \cos\theta$

$y = \sin\theta$

coordinatizes the unit circle

$x^2 + y^2 = 1.$



if $f' = \pm g$

& $g' = \mp f$.

Then (f, g) are

cos, sin up to

<u>a translation</u>

$$(f(\theta), g(\theta)) = (\cos(\theta + \theta_0), \sin(\theta + \theta_0))$$

$$' = \frac{d}{d\theta}.$$