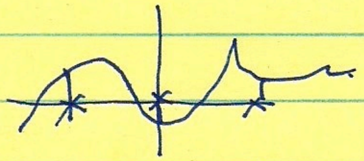
(1)

Lecture 2 19A
Jan 10, '18.

<u>Bring</u> : codes, Big chalk.
ruler!

ask Salvador's help!

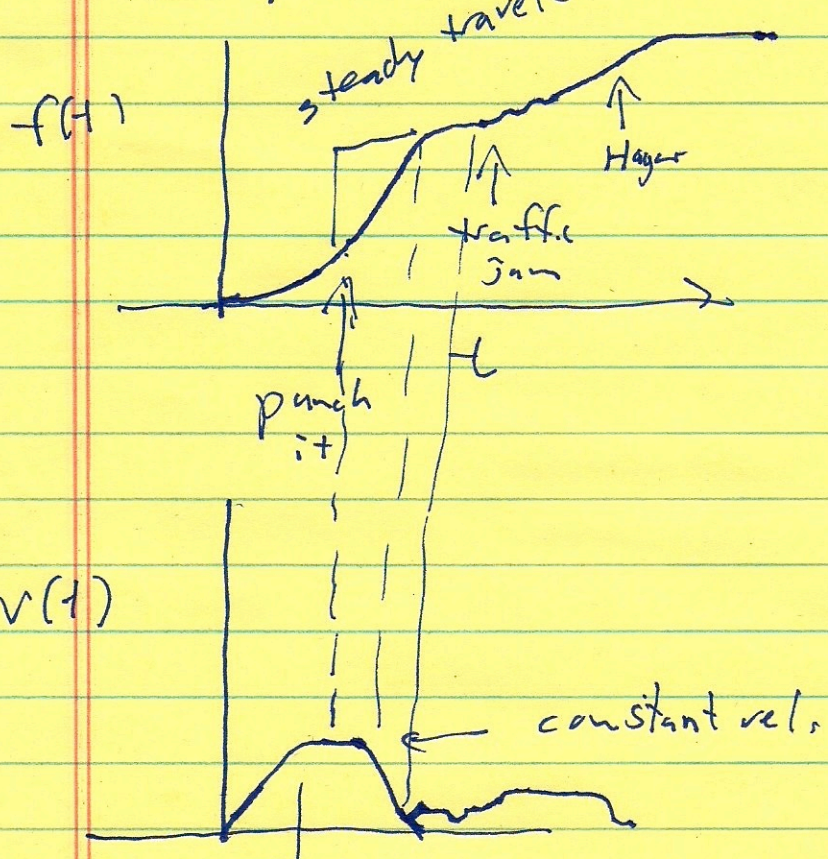
(1) Quiz :



Anyone Nive today.

(2) Rate of change ; distance $\longleftrightarrow$ vel.

Story



f(t) = dist from house at time t.
units L = length

$\frac{df}{dt} = v(t)$
= speed at time t.

You have seen These ~~some~~ you nev try.
ODOMETER : integrates
SPEEDOMETER : differentiates

Play with graph:

what if ~~I~~ started 10 min later?

~~What~~ How do graphs change?

what if ~~in~~ his car is parked ~~B~~ a mile from home?

~~What doe~~

Our $f(t)$ is monotonic $\uparrow$

What would it mean if $\downarrow$?

Okay the map $f \mapsto f'$

or $\frac{d}{dx}$.

Maps $\mathcal{F}$ = 'nice' function on $\mathbb{R}$ to $\mathcal{F}$.

$$\mathcal{F} \xrightarrow{\frac{d}{dx}} \mathcal{F}.$$

An operator or "functional"

Function of functions

Basic Properties. linear

$$(f+g)' = f' + g'$$

~~or~~ $(cf)' = cf'$, c const.

This + power rule is enough to compute the derivative of polynomials.

Exer $\frac{d}{dx}\left(x^4+\frac{1}{3}x^3+7x^2+12\right)$

$= ?$

Do!

---

what other algebraic rules holds for $\frac{d}{dx}$?

• ??. Audience ...

$(fg)' = fg' + f'g.$ Liebnitz ~ product.

Eg. $x^5 = \underbrace{(x^2)}_{f}\underbrace{(x^3)}_{g}$

We know:

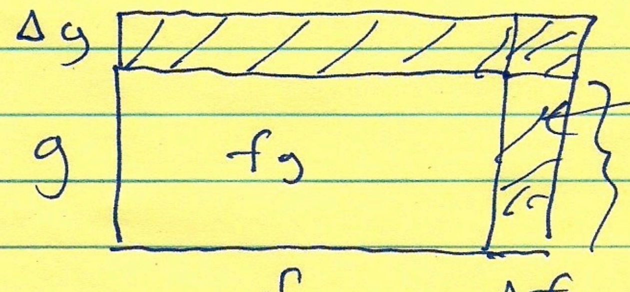
$$\frac{d}{dx} x^5 = 5x^4$$

$$\frac{d}{dx} x^2 = 2x; \quad \frac{d}{dx} x^3 = 3x^2.$$

$$f'g + fg' = 2x \cdot x^3 + x^2 \cdot (3x^2)$$
$$= 2x^4 + 3x^4$$
$$= 5x^4$$

OK! Math consistent!

Picture for product rule:



$$\Delta(fg) = (\Delta f)g + f(\Delta g) + (\Delta f)(\Delta g)$$

¿ What's this Δf stuff ??

$$\Delta f = \boxed{f(x + \Delta x) - f(x) \approx f'(x)\Delta x}$$

L. 2

5

Similarly
$$\Delta g := g(x+\Delta x) - g(x)$$
$$\simeq g'(x)\,\Delta x.$$

linear approx.

gets better & better as $\Delta x \to 0$.

Mention: ~~da~~ HW due before every lecture starting Wed, just 1 problem.

☆ then $\simeq$ 10 problems due every Tuesday.

Pf of Power law from Product rule: By Induction

aside on Induction.

Statement: P_n: The derivative of x^n is nx^{n-1}.

Strategy: Prove P_1

Show $P_n \Rightarrow P_{n+1}$, $\forall n$.

P_1 ? Why true?

Next $P_n \Rightarrow P_{n+1}$ "The Inductive Step"

P_n: say we know $\frac{d}{dx} x^n = n x^{n-1}$ (P_n)

for some integer n.

P_{n+1}: We will prove: $\frac{d}{dx} x^{n+1} = (n+1)x^n$

(P_{n+1})

well ...

$$x^{n+1} = \underset{f}{x}\ \underset{g}{x^n}.$$

so

$$\begin{aligned}(x^{n+1})' &= (x)' x^n + x(x^n)' \\ &= 1 \cdot x^n + x(n x^{n-1}) \\ &= x^n + n x^n \\ &= (n+1) x^n.\end{aligned}$$

Yay! QED.

Back to the grind...

What other functions are true besides polynomials?

?? Audience. ... $p(x)$

e^x, $\sin x$, $\cos x$, ...

anything else?

$x^{1/2}$ $x^{p/q}$ x^2.

$\sin^{-1}$ $\log$.

can get these using these & chain rule next week.

$\boxed{\text{Bolzano-Weierstrass. (?),}}$

To e^x, $= y(x)$

Best def. it is the fn s.t.

$\boxed{y' = y}$ and $y(0) = 1$.

If time: ---
start in! (e^x, Taylor series...)

Try Find a poly $p(x)$
s.t. $\boxed{p'(x) = p(x)}$ & $p(0) = 1$.

so: $p(x) = 1 + a_1 x + a_2 x^2 + \cdots$
&
class → $p'(x) =$ _ _ _

Equate coeff:

$p'(x) = a_1 + 2a_2 x +$

$a_1 = 1$
$2a_2 = a_1$ so $a_2 = \frac{1}{2}$.

so far
$p(x) = 1 + x + \frac{1}{2}x^2 + a_3 x^3 + \cdots$

$3a_3 = a_2$, $a_3 = \frac{1}{3 \cdot 2}$.
$\vdots$
$n \cdot a_n = a_{n-1}$

By induc! $a_n = ?$

You have found the power series for e^x.

$\boxed{\text{There is also a number } e \simeq 2.718.}$

s.t.

" $e^2 = e^2$ " ie $\cancel{f}(2) = e \cdot e$

$\cancel{f}(3) = e \cdot e \cdot e$

$\vdots$

$y(n) = \underbrace{e \cdots e}$.

On to Salvador Guerrero!

for

Friday