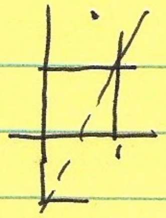
Bring a Ruler

Class 1

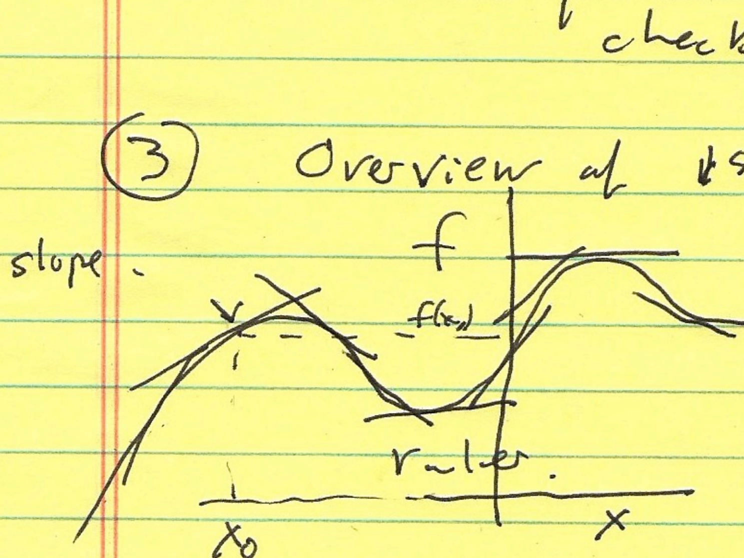
① Quiz.



thru (1,1)
slope 2.
- draw
- write down eqn.

② Introductions: to 3 neighbors
hand quiz over.
check.

③ Overview of 1st 2 qtrs of calc
19A, B.



19A: Break up a curve into infinitely many straight lines.

19B: Putting all these straight lines back together then form the original curve (graph)

Def: The slope m of the tangent line to the graph of f at the point $(x_0, f(x_0))$ is the derivative of f at x_0.

Notation: $f'(x_0)$ or $\left.\frac{df}{dx}\right|_{x_0}$ or $\frac{df}{dx}(x_0)$.

Save.

Def. The derivative of $f(x)$ at the point $x = x_0$ is the slope of the tangent line to the graph of f at $(x_0, f(x_0))$.

Notation for derivative:

$$f'(x_0) \quad \text{or} \quad \frac{df}{dx}(x_0) \quad \text{or} \quad \left.\frac{df}{dx}\right|_{x_0}.$$

As we let x_0 vary the slope varies, we get a new function $f'(x)$, or $\frac{df}{dx}(x)$.

19A: Computing & Using derivatives.

19B: Putting all the little lines together to recover the ~~de~~ function. Called "Integration"

Fund. Thm of Calculus: Asserts integration & differentiation are inverse operations:

$$\text{"} f(x) = \int f'(x)\,dx \text{"}$$

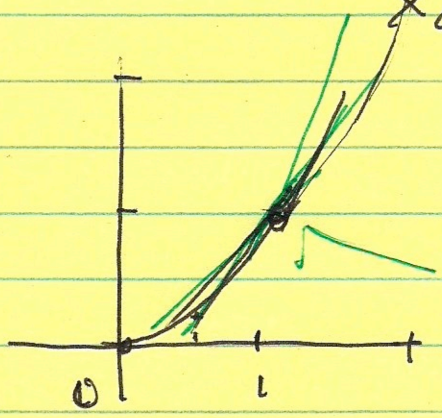
integral derivative.

Start In

(4) How to compute the derivative.

Example $f(x) = x^2$.

$x_0 = 1$.



with ruler.

Does any know?

$f'(1) = ??$

slope of Secants: line from

i) $(1,1)$ to $(2,4)$

ii) $(1,1)$ to $(1\tfrac{1}{2}, (1\tfrac{1}{2})^3)$

iii) $(1,1)$ to $(1\tfrac{1}{3}, (1+\tfrac{1}{3})^2)$

etc.

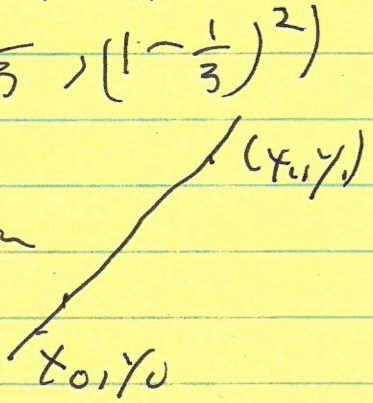
⋮

$(1,1)$ to $(\tfrac{1}{2}, (\tfrac{1}{4}))$

$(1,1)$ to $(1-\tfrac{1}{3}, (1-\tfrac{1}{3})^2)$

⋮

$\dfrac{y_1 - y_0}{x_1 - x_0}$ = slope of line from



ies: $\dfrac{4-1}{1-1} = 3$; $\dfrac{(1+\frac{1}{2})^2 - 1}{\frac{1}{2}}$ ...

Python program ?

~~Case~~ $x_1 = 1 + \frac{1}{10}$.

$y_1 = x_1^2 = (1+\frac{1}{10})^2 = 1 + \frac{2}{10} + \frac{1}{100}$

$y_1 - y_0 = \frac{2}{10} + \frac{1}{100}$.

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{\frac{2}{10} + \frac{1}{100}}{\frac{1}{10}} = 10\left(\frac{2}{10} + \frac{1}{100}\right)$$

$$= 2 + \frac{1}{10}$$

$$= 2.1$$

$$\approx 2.$$

General h as Δx for a very small variable number.

$x_h = 1 + h$.

$y_h = f(x_h) = (1+h)^2$.

$x_h - x_0 = h$

$$\frac{\Delta y}{\Delta x} = \frac{y_h - y_0}{x_h - x_0}$$

$$= \frac{(1+h)^2 - 1}{h} = \frac{(1 + 2h + h^2 - 1)}{h}.$$

$$= \frac{2h + h^2}{h} = \boxed{2 + h}$$

Key step.

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(5)

Now let $h \to 0$.

$f'(1) = 2.$

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(5): Go over Syllabus.

Main Points,

☆ Read tomorrows lesson ~~today~~ before tomorrow

Spend 5-10 hrs of HW / week.

Make a friend taking the class & study & work problems with them.

Try other sources.

Grade Distribution

Reading for Wed: 3.4

for today 3.1, 3.2.

Back to derivatives

if $y = 3x+1$

Ask what is $\frac{dy}{dx}$? ← why?

$\boxed{\text{Go back to definition.}}$

Back to.

if $f(x) = x^2$

we showed $f'(1) = 2$.

$f'(2) = ?$

$f'(3) = ?$

$f'(\frac{1}{2}) = -$

$f'(x) = ?$

Why?

Do some algebra!

$$f'(x_0) = \lim_{h \to 0} \frac{(x_0+h)^2 - x_0^2}{h}$$

Approx derivative.

$$\frac{\Delta f}{\Delta x} = \qquad = \frac{x_0^2 + 2x_0h + h^2 - x_0^2}{h}$$

19A lecture 1, Jan 8, '18

$$= \frac{2x_0 h + h^2}{h} = 2x_0 + h$$

Now let $h \to 0$:

get: $2x_0$.

Limits??

~~Read~~ chapter 2,

or watch the ruler move along the secants.

Other derivatives.

$f(x) = x^3$, $f'(x) =$

$f(x) = x^4$, $f'(x) =$

⋮

$f(x) = x^n$, $f'(x) =$.

"Power rule"

Then the algebraic properties of derivative

Derivative as an Operator.

$$\frac{d}{dx}$$

Functions $\longrightarrow$ Functions

$$f \longmapsto f'$$

$$\text{or } f(x) \longmapsto \frac{df}{dx}(x),$$

$\boxed{\text{Hsiang's Box}}$