A missing function:

$f(x)$ such that $f'(x) = \frac{1}{x}$.

$$\frac{d}{dx}\Big\downarrow \quad \begin{array}{cccccc} -1x^{-1} & ? & x & \frac{1}{2}x^2 & \frac{1}{3}x^3 & \frac{1}{4}x^4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \quad \cdots \\ x^{-2}, & x^{-1} & 1 = x^0, & x, & x^2, & x^3 \end{array}$$

?

$f(x) = \ln(x) := \log_e x$ ← satisfies $\boxed{f'(x) = \frac{1}{x}}$

$= $ inverse function of ~~the~~ e^x

Recall: $10^3 = 1000 \iff \log_{10} 1000 = 3$

$$2^x = y \iff \log_2 y = x.$$

so

$$e^x = y \iff \log_e y = x$$

or $e^{\log_e x} = x$ & $x^{\log_e Y} = x$.

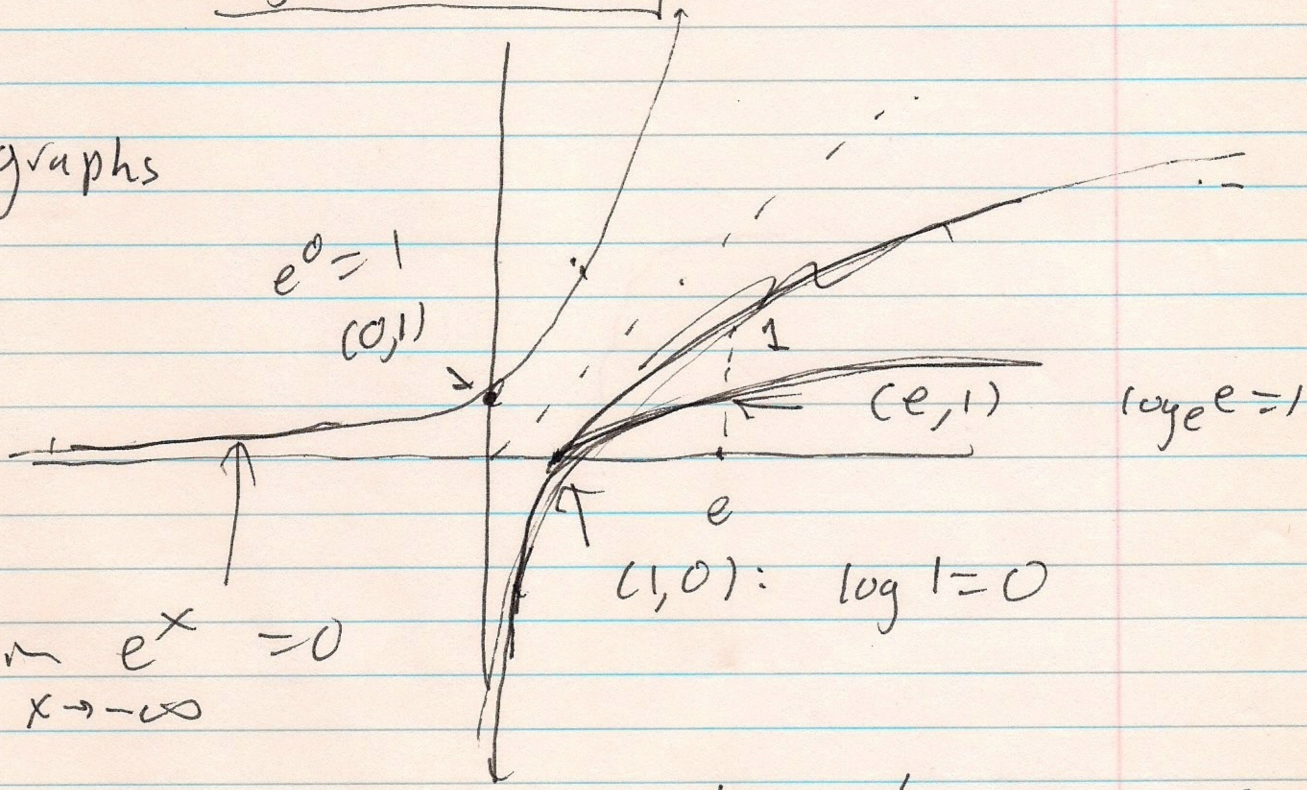
$e^{g(x)} = x \qquad g(x) = \log_e x.$

$\frac{d}{dx}: \quad (e^{g(x)})\, g'(x) = 1$

$(x)\, g'(x) = 1$

$$\boxed{g'(x) = \frac{1}{x}.}$$

graphs



$\lim_{x \to -\infty} e^x = 0$

$\left(e^{-1000} \simeq 0\right)$

$\lim_{x \to 0} \log x = -\infty$

To compute:

$$\frac{d}{dt} 2^t$$

hint: $2 = e^{(\log 2)}$

$$\frac{d}{dx} x^x$$

hint $x = e^?$

HW?

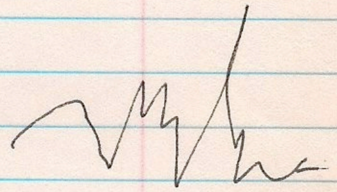
<u>Questions</u>?

Applications of log.

$$\frac{d}{dt}\log f(t) = \frac{f'(t)}{f(t)}$$

$$= \frac{\text{rate of change of } f}{\text{size of } f}.$$

<u>Earthquakes.</u> (wiki)



$$\text{Richter} = \log_{10} A - C$$

$$= \log_{10}\left(\frac{A}{A_0}\right)$$

$C = \log_{10} A_0$

various empirical formula.

so:

$A \to 10A$ $\qquad \log_{10}(10A) = 1 + \log_{10} A$

$R \to R+1$

$A \sim \text{Energy}$

$$\frac{dR}{dA} = ?$$

$$\log_{10} A = ?(\log_e A)$$

$$\log_{10} A = y \Rightarrow A = 10^y$$

$$A = e^w$$

where $10 = e^{\log_e 10}$.

$$\Rightarrow 10^y = e^{(\log_e 10) y}$$

$$w = (\log_e 10) \log_{10} A.$$

So

$$\log_{10} A = (\log_e 10) \log_e A.$$

$$\frac{d}{dA} \log_{10} A = k \cdot \frac{1}{A} \quad ; \quad \underline{k = \log_e 10}$$

[~~[illegible]~~ ?]

§1.6, Active Calculus.

f''; convexity; or from text in ch. 4.