Mean Value Theorem.

Oakland: ~~Is~~ 80 miles from here. You drove there in 1 hour.

<u>Then</u>: at some point you were going 80 mph (& so deserved a ticket)!

<u>Math speak.</u>

Thm
f continuous & differentiable on $[a,b]$. Suppose.

$$v = \frac{f(b) - f(a)}{b-a}.$$

Then $\exists\ x_*$, with $a \le x_* < b$.
&
$$f'(x_*) = v$$

<u>Do</u>: Web Assign $\sqrt{x}$
$a = 4$, $b = 100$
problem. (of finding x_*)

Case where $f(a) = f(b)$
called "Rolle's theorem"

asserts: $\exists\, x_*$, $a \leq x_* \leq b$
& $f'(x_*) = 0$.

Pf.

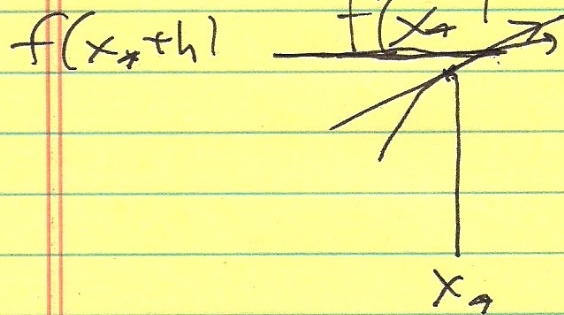
First prove: If f has a max or min at x_* & f is differentiable at x_* then $f'(x_*) = 0$.

Pf. By contraposition.
Say f is differentiable at x_* & $f'(x_*) \neq 0$. Then f is not a max or min

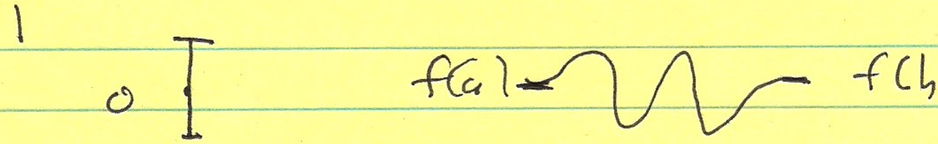
Pf: Say $f'(x_*) = c > 0$.

Then $f(x_* + h) \simeq f(x_*) + ch + \cdots$
so for $h > 0$, $f(x_* + h) > f(x_*)$.

Intuitive proof:



Now By IFT $f([a,b])$ = interval



has loc. max & min in between a & b.

$\therefore$ pts x_* s.t. $f'(x_*)=0$.

General case of Mean Value Theorem follows from Rolle's.

Given: $f(x)$

$$f(a) = y_a.$$
$$f(b) = y_b.$$

$$v = \frac{f(b) - f(a)}{b - a}.$$

Form: $g(x) = f(x) - v \cdot (x - a).$

Then: $g(a) = f(a)$

$$g(b) = f(b) - v\,(b - a)$$
$$= f(b) - (f(b) - f(a))$$
$$= f(a) = g(a)$$

& $g'(x) = f'(x) - v.$

Apply Rolle's: $\rightarrow g'(x_*) = 0$

of $\boxed{f'(x_*) = v}$