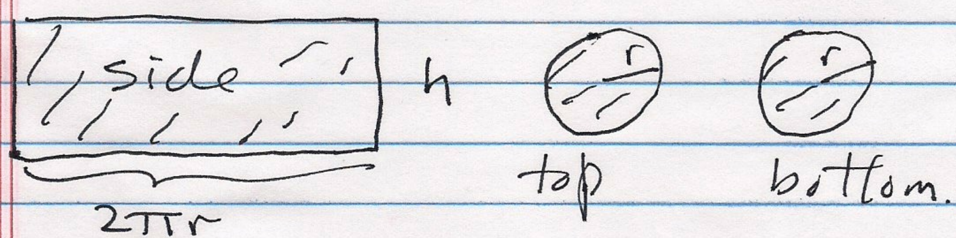
Way 1

Fix the surface area of a right circular cylinder. Let r be the radius & h the height of the cylinder. ¿What "aspect ratio" $h:r$ maximizes the volume V for this given surface area S? Take the surface area to include the tops & bottoms of the "tin can".



$$S = 2\pi r h + \pi r^2 + \pi r^2$$
$$= 2\pi r h + 2\pi r^2.$$

$$V = \pi r^2 h$$

Way 1: Solve for h in the S-eqn. Plug result into V to get an eqn for $V = V(r)$, indep of r. Now maximize by setting derivative to be zero:

Way 1

well

$$S = 2\pi r h + 2\pi r^2$$

$$S - 2\pi r^2 = 2\pi r h.$$

$$\frac{S - 2\pi r^2}{2\pi r} = h.$$

So

$$h = \frac{S}{2\pi r} - r$$

Plugging into V:

$$V = \pi r^2 h = \pi r^2 \left(\frac{S}{2\pi r} - r\right)$$

$$= \frac{\pi}{2\pi} S \frac{r^2}{r} - \pi r^3.$$

$$= \frac{S}{2} r - \pi r^3.$$

[check units!]

So $V''(r) = \frac{S}{2} - 3\pi r^2.$

Set $= 0.$

$$\Rightarrow 3\pi r^2 = \frac{S}{2}.$$

~~Last~~ Now from here compute

$$\frac{h}{r}.$$

Use $\frac{h}{r} = \frac{hr}{r^2}$ Way II

well:

$$r^2 = \frac{1}{3\pi}\frac{S}{2} = \frac{S}{6\pi}$$

$$rh = r\left(\frac{S}{2\pi r} - r\right)$$

$$= \frac{S}{2\pi} - r^2$$

$$= \frac{S}{2\pi} - \frac{S}{6\pi}$$

Since $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

get

$$rh = \frac{1}{3\pi} S.$$

So

$$\frac{h}{r} = \cancel{\frac{1}{3\pi}\frac{S}{2}} \frac{\frac{1}{3\pi} S}{\frac{1}{2}\frac{1}{3\pi} S.}$$

$$= 2$$

or

so: $h : r = 2$

or $h = 2r.$

In other words

$$\boxed{\text{Height} = \text{Diameter}}$$

Way 2 Implicitly differentiate the S eqn to find $\frac{dh}{dr}$. Use this in V to find a simple eqn for h/r coming from $V'(r)=0$

$S = \text{const.}$ so $\frac{dS}{dr} = 0.$

But

$S = 2\pi r h + 2\pi r^2$ which defines $h = h(r)$

So

$$\frac{dS}{dr} = 2\pi\left(r\frac{dh}{dr} + 1 \cdot h\right) + 4\pi r.$$
$$= 0.$$

$$\Rightarrow r\frac{dh}{dr} + h + 2r = 0.$$

$$\frac{dh}{dr} = -\frac{1}{r}(h+2r) = -\frac{h}{r} - 2.$$

Now use this to compute:

$$\frac{d}{dr}V = \frac{d}{dr}\pi r^2 h = 2\pi r h + \pi r^2 \frac{dh}{dr}$$

Set $\frac{dV}{dr} = 0$. Get:

$$0 = 2\pi r h + \pi r^2 \frac{dh}{dr}$$

Way 2 ctnd

Plugging in what we found out about $\frac{dh}{dr}$ & ~~dividing~~ dividing by π:

$$0 = 2rh + r^2\left(-\frac{h}{r} - 2\right)$$

Divide by r^2:

$$0 = 2\frac{h}{r} - \left(\frac{h}{r} + 2\right)$$

or

$$0 = \frac{h}{r} - 2$$

Again:

$$h = 2r$$

Height = Diameter.