2/5/18, Midterm. A

Midterm Curve
...
Pickups

Common weaknesses:

lack of understanding of exponents.

Eg in one version had to estimate $\sqrt[5]{31}$

a) what integer is it close to

Try: $1^5 = 1$; $\sqrt[5]{1} = 1$
$2^5 = 32$; $\sqrt[5]{32} = 2$.

$31 \simeq 32$. Indeed $31 = 32 - 1$

Set $f(x) = x^{1/5}$; $f(31) \simeq f(32) + f'(32)(-1)$
linear approx.

$f(32) = 2$; $f'(x) = \frac{1}{5} x^{\frac{1}{5}-1}$
$= \frac{1}{5} x^{-4/5}$

$f'(32) = \frac{1}{5}(32)^{-4/5}$

various embarrasing answers:

$32^{-4/5}$ = .. $\frac{1}{1,000}$; $-\frac{4}{25}\frac{1}{32}$.

let's try in good faith

B

$b > 0.$

$b^1 = b.$
$b^2 = b \cdot b.$
$b^3 = b \cdot b \cdot b$

$$b^m \cdot b^n = \underbrace{(b \cdot \cdots \cdot b)}_{m} \cdot \underbrace{(b \cdot \cdots \cdot b)}_{n}$$

$$= b^{m+n}$$

We require this rule:

$$b^m \cdot b^n = b^{m+n}.$$

For all numbers. Only thing you need to remember.

Then what is b^0?

Well: $b^m \cdot b^0 = b^{m+0} = b^m.$

so $b^0 (b^m) = b^m \Rightarrow b^0 = 1$

What is b^{-1}?

$$b^1 \cdot b^{-1} = b^{1+-1} = b^0 = 1.$$

or $b \cdot b^{-1} = 1$

$$\Rightarrow b^{-1} = \frac{1}{b}$$

C

How about

$$(b^m)^n \; ?$$

Well .. ~~m~~

$$(b^m)^n = \underbrace{b^m \cdot b^m \cdot \dots \cdot b^m}_{n \text{ times}}$$

$$= \underbrace{\underbrace{(b \cdot \dots \cdot b)}_{m} \underbrace{(b \cdot \dots \cdot b)}_{m} \cdot \dots \cdot \underbrace{(b \cdot \dots \cdot b)}_{m}}_{n.}$$

$$= b^{m \cdot n}$$

& Also holds for all numbers.!

We know $b^{1/5} = 2$, when $b = 32$.

What is $b^{-4/5}$?

Well: $-\frac{4}{5} = \frac{1}{5} \cdot 4 \cdot -1$

So $b^{-4/5} = \left(\left(b^{1/5}\right)^4\right)^{-1}$

$$= (2^4)^{-1}$$

$$= \frac{1}{16}.$$

D

on the last problem.

A) graph $t\sin t. = g(t)$

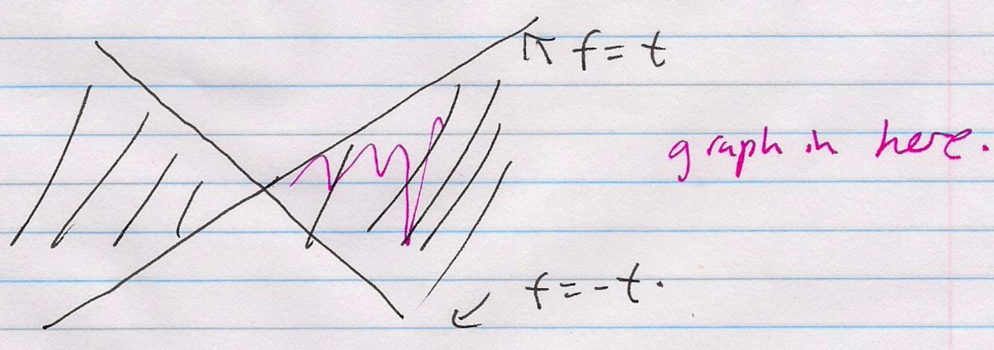
Key: $\sin t$ is bounded:

$|\sin t| \le 1$

also: sin is odd
t is odd
so $t\sin t$ is even.

recall: odd: $f(-t) = -f(t)$
even $f(-t) = +f(-t)$.

Set of (t, f) in (t, f) plane with $|f| \le t$:

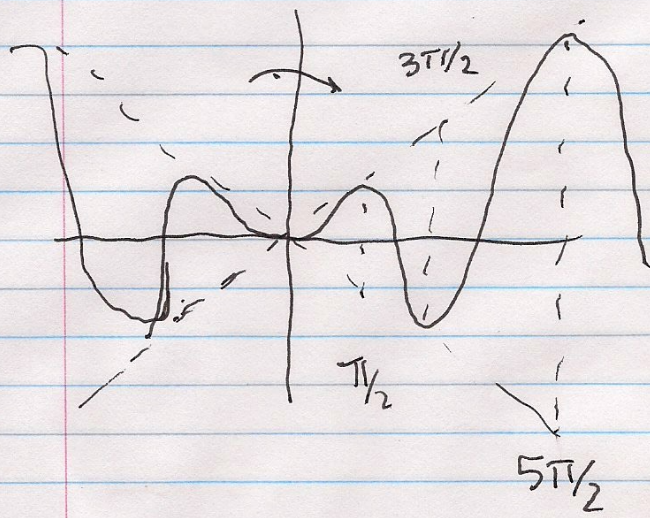


Every $\pi/2$, $\pi/2 + 2\pi$, $\pi/2 + 4\pi$ $\sin t$ maxes out at 1

Every $3\pi/2$, $3\pi/2 + 2\pi$: Mins out at -1 :

E

A) cont'd : so



B) Is it possible that

$$g'(t) \ge t+1, \quad \text{for } 1 \le t \le 11 \,?$$

NO

well $g'(t) = \frac{d}{dt}(t \sin t)$

$$= t\frac{d}{dt}(\sin t) + \left(\frac{dt}{dt}\right)\sin t$$

$$= t\cos t + \sin t.$$

Now $|\cos t| \le 1$, $|\sin t| \le 1$

so $|g'(t)| \le |t||\cos t| + |\sin t| \le t+1$

~~On~~ To get equality need:
$\cos t = 1$ & $\sin t = 1$.

Cannot happen for same t's!