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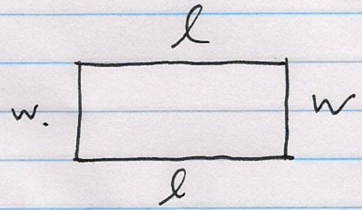
Minimizing. § 4.2, 4.7

Constrained minimization

"Applied optimization"

Example problem.

40' of linear feet of wood. What are the dimensions of the rectangle of maximum area?



$l + w + l + w = 40.$

Maximize $l \cdot w$.

Guess??

Of course! Square.

Verifying: $2l + 2w = 40$

so $w = 20 - l.$

$$\text{Area} = lw = l(20 - l)$$
$$= 20l - l^2.$$

$\frac{dA}{dl} = 0$ at max:

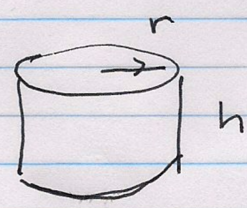
$$20 - 2l = 0.$$

$$l = 0.$$

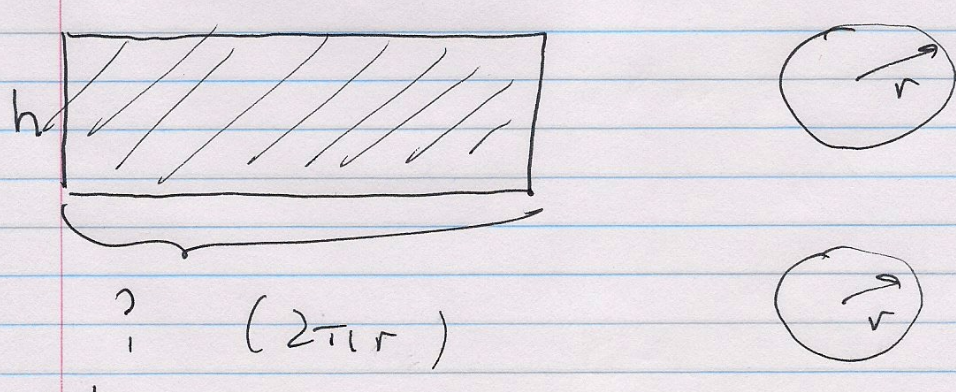
box is 10×10.

Less obvious (Tin Can!)

build a cylinder, with caps.
(a "keg" or "winebarrel"



~~of ma~~ containing the maximum possible volume, for a given total surface area S.



Wha

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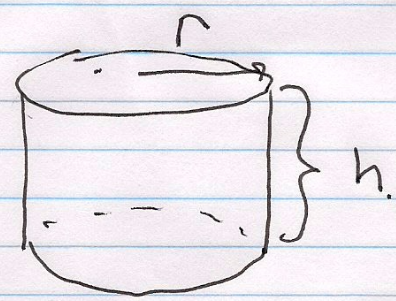
What is the aspect ratio

$h:r$? ie $\frac{h}{r}$?

of the optimal

tin can ?



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$$S = 2\pi r h + \pi r^2 + \pi r^2$$

$$= 2\pi r(h+r).$$

$$V = \pi r^2 h.$$

Hmm: $\pi r^2 =$

$$S = 2\pi r h + 2\pi r^2$$

$$2\pi r h = S - 2\pi r^2.$$

$$h = \frac{S}{2\pi r} - r.$$

So:

$$V = \pi r^2\left(\frac{S}{2\pi r} - r\right)$$

$$= \cancel{\pi S}\ \frac{Sr}{\cancel{\ \ }} - \pi r^3.$$

$$\frac{dV}{dr} = \frac{S}{\cancel{\ \ \ \ }} - 3\pi r^2. = 0.$$

$$\Rightarrow r^2 = \frac{S}{3\pi}$$

so $2\pi r^2 = \frac{2}{3} S$

$$2\pi r h = \frac{1}{3} S$$

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& $\frac{h}{r} = \frac{2\pi r h}{2\pi r^2} = \frac{\cancel{1}/3\, S}{2/3\, S}$

$= \frac{1}{2}.$

Turn to case with bounds

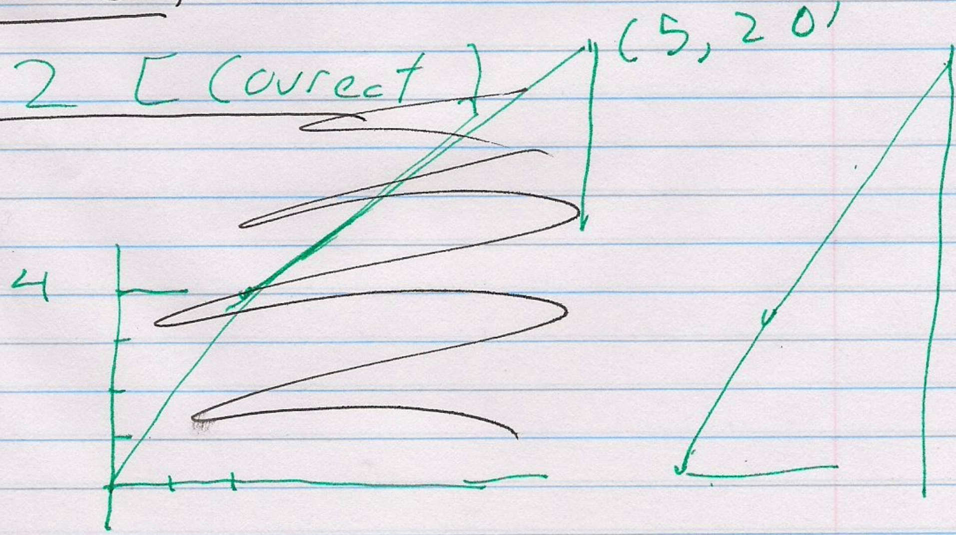
Minimize $2x = f(x)$

subject to $2 \leq x \leq 5$

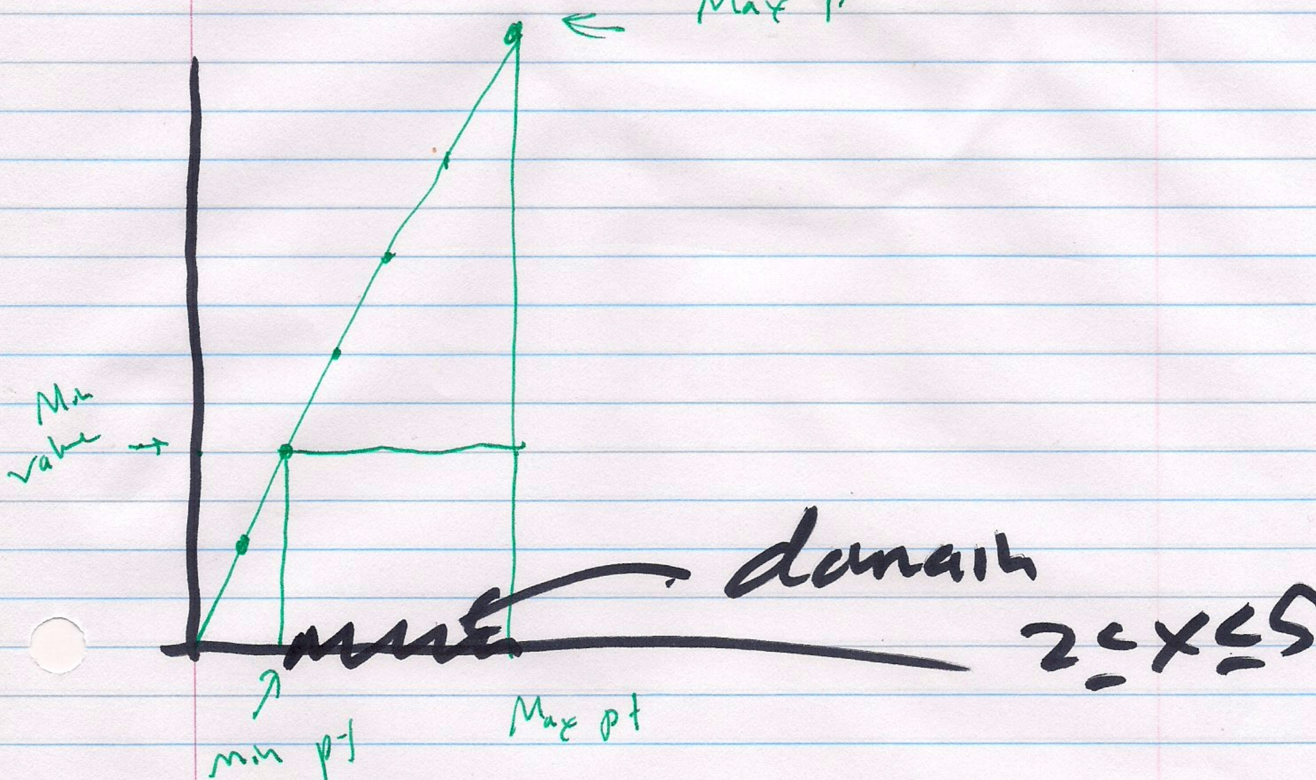
Soln 1: $\frac{df}{dx} = 2 \neq 0$. No min or max

WRONG!

Soln 2 [Correct]



Better:



general algorithm:
for f differentiable
constrained to $a \le x \le b$.

Check: $f(a)$, $f(b)$
& any critical pts $f'(x) = 0$
with critical values $f(x)$
for $a < x < b$

Get, typically, a finite list of numbers. Find the least.

Text (4.2)

calls any of these

points a, b, & x with $f'(x)=0$ & $a<x<b$

"extremal points

with corresponding values

$f(a)$, $f(b)$, $f(x)$ where $f'(x)=0$ & $a<x<b$.

"extremal values"

More usual: interior points are "critical points

& other points are "endpoints"

Now Go To

"My HW for Feb 11"