Then to 5.5 exer.

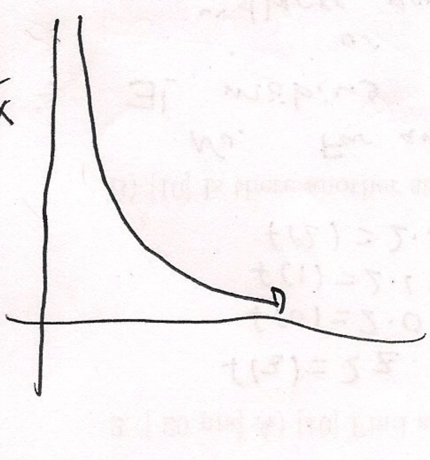
Sketch the curve, identifying local maxima, minima, inflection points & intercepts, asymptotes & behaviour near $\pm\infty$.

$$y = x + \frac{1}{x}$$

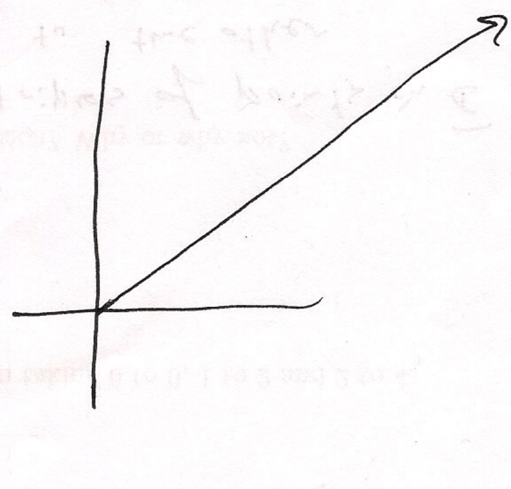
x beats $\frac{1}{x}$ for $x > 0$.

$\frac{1}{x}$ beats x for $x < 1$

$\frac{1}{x}$

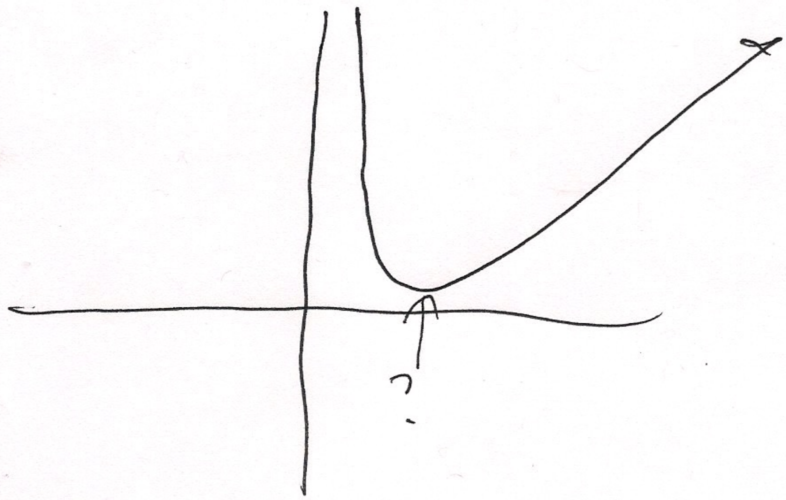


x



+

roughly!



Does it ~~ever~~ cross $y=0$?

$$\left(x+\frac{1}{x}\right)' = 1-\frac{1}{x^2}$$

$$y'=0 \Leftrightarrow 1-\frac{1}{x^2}=0 \Leftrightarrow x^2=1$$

$$x=\pm 1$$

$? = 1$. Min value? $1+\frac{1}{1}=2$

inflection points?

$$y'' = 1' - \left(\frac{1}{x^2}\right)' = 0-(-2)x^{-3}$$

$$= 2/x^3 > 0$$

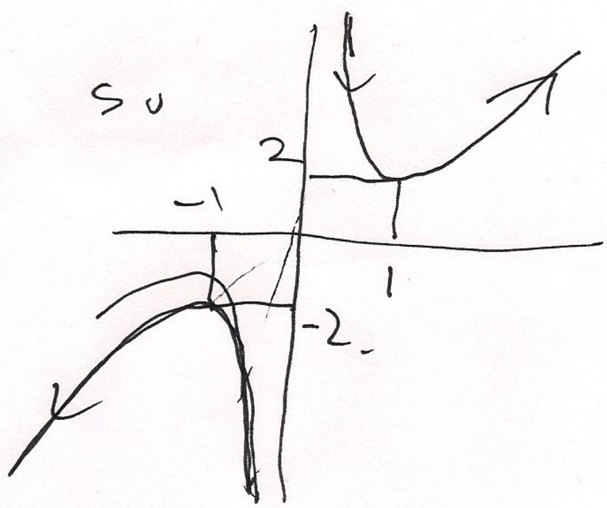
convex!

What about for $x < 0$?

$y(x)$ is odd since $x \mapsto x$
& $x \mapsto \frac{1}{x}$
are odd

& for <u>functions</u> odd + odd = odd
(not for numbers)

so



$(x,y) \in$ graph

$\Leftrightarrow (-x,-y) \in$ graph

since
$y(-x) = -y(x)$

A function that comes in astronomy (Celestial mechanics) & quantum mechanics:

$$f(r) = \frac{L^2}{r^2} - \frac{\mu}{r} \quad , L, \mu > 0$$

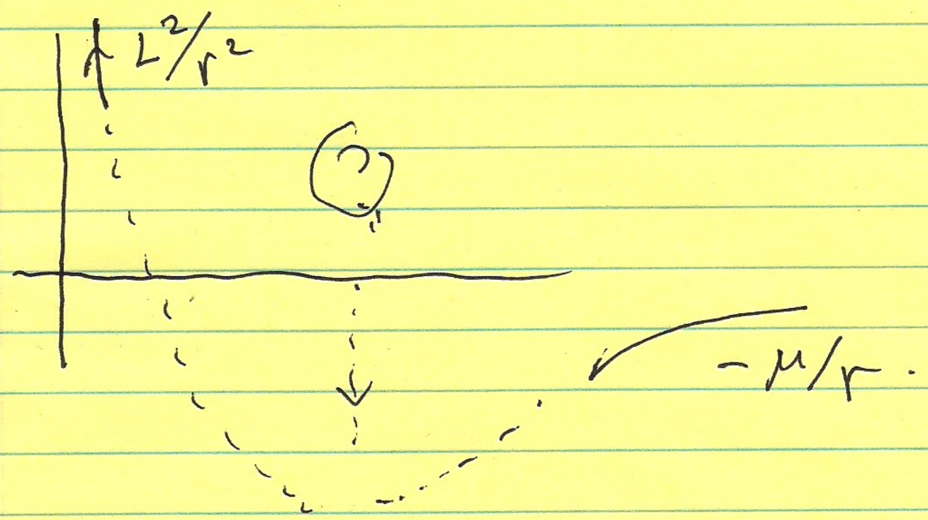
$$r > 0$$

for $r < 1$ $\frac{1}{r^2}$ beats $\frac{1}{r}$.

for $r > 1$ $\frac{1}{r}$ beats $\frac{1}{r^2}$

(Why?)

so:



$f'(r) = ?$

To make algebra easier, less mistakes, let us set

$\mu = 1.$

$$f(r) = \frac{L^2}{r^2} - \frac{1}{r}.$$

$$f'(r) = \frac{-2L^2}{r^3} + \frac{1}{r^2}$$

$$f''(r) = (-3)\frac{(-2L^2)}{r^4} + (-2)\frac{1}{r^3}$$

$$= \frac{6L^2}{r^4} - \frac{2}{r^3}.$$

<u>Min:</u> $f'(r) = 0. \iff \frac{-2L^2}{r^3} + \frac{1}{r^2} = 0$

$$\iff \frac{1}{r^2} = \frac{2L^2}{r^3}$$

$$\iff r = 2L^2$$

Value at min?

$$f(r) = \frac{L^2}{r^2} - \frac{1}{r}$$

But at min $\frac{2L^2}{r^3} = \frac{1}{r^2}$ $\times r$.

so $\frac{2L^2}{r^2} = \frac{1}{r}$.

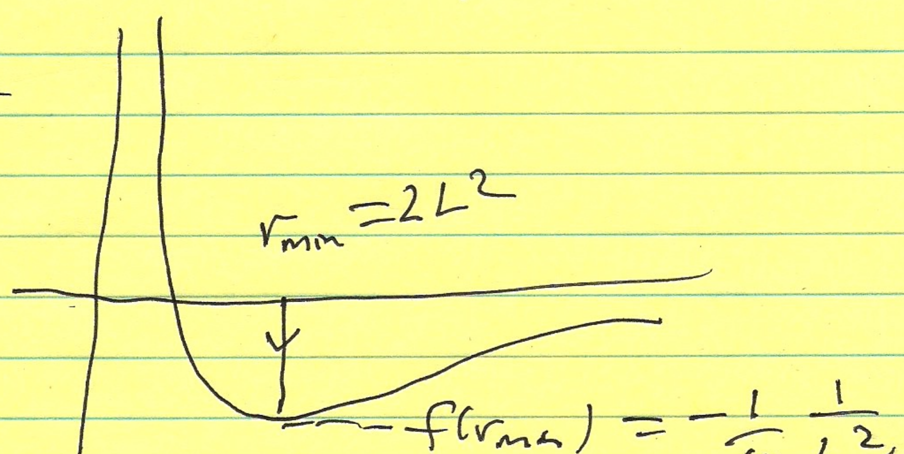
so $f(r_{min}) = \frac{L^2}{r_{min}^2} - \frac{2L^2}{r_{min}^2}$

$= -\frac{L^2}{r_{min}^2}$.

& $r_{min} = 2L^2$, so $r_{min}^2 = 4L^4$

& $f(r_{min}) = -\frac{L^2}{4L^4} = -\frac{1}{4}\frac{1}{L^2}$.

So far



Inflection points?
Where, to eye?

$f''(r) = 0.$

$$\Leftrightarrow \quad \frac{6L^2}{r^4} - \frac{2}{r^3} = 0$$

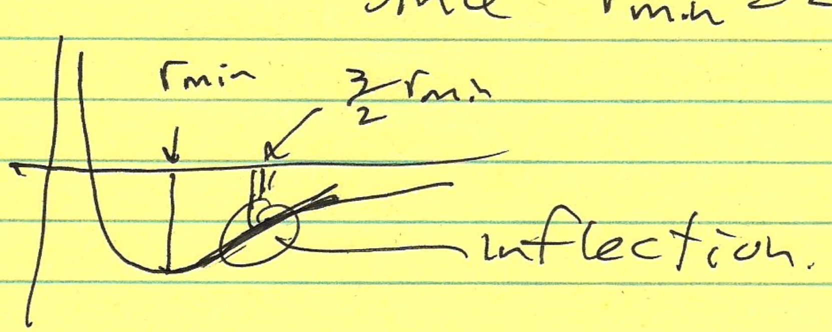
$$\frac{2}{r^3} = \frac{6L^2}{r^4}$$

$\times r^4$: $\quad 2r = 6L^2$

$$\boxed{r = 3L^2}$$

$$= \frac{3}{2} r_{min}$$

since $r_{min} = 2L^2$.



Meaning of:

Classical:

r_{min} = location of circular orbit for a given angular momentum L.

Quantum:

$f(r_{min}) \sim \underline{\text{energy}} \sim \frac{1}{L^2}$

$L = \ell(\ell+1)$

so energy levels $\sim \frac{1}{\ell^2}$.

Spectrum hydrogen.

Error ~~errors~~

$f'(r) = \frac{-2L^2}{r^3} + \frac{\mu}{r^2}$.

How'd I find??

$f'(r) = 0 \iff \frac{2L^2}{r^3} = \frac{\mu}{r^2}$

Algebra Error

$\iff 2L^2 = \mu r$

Algebra error!

$\Downarrow$

$r = \frac{\mu}{2L^2}$.

One min! a global min,

f

Any maxes?? (No!)

Inflection pt.? Look first!

$f'' = (-3)\left(\frac{-2L^2}{r^4}\right) + (-2)\frac{\mu}{r^3}$

$= \frac{6L^2}{r^4} - \frac{2\mu}{r^3}$.

$f''(r) = 0 \iff \frac{6L^2}{r^4} = \frac{2\mu}{r^3}$

$\iff 6L^2 = 2\mu r \iff r = \frac{2\mu}{6L^2} = \frac{1}{3}\frac{\mu}{L^2}$.

discovered here