

symbol for the operation: *differentiation* with respect to x .

In terms of such a system of notations, we summarize the basic laws of differentiation and integration as follows:

$$(i) \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\text{or } d(f(x) + g(x)) = df(x) + dg(x).$$

$$(ii) \frac{d}{dx}(c \cdot f(x)) = c \frac{d}{dx}f(x), \quad c = \text{constant}$$

$$\text{or } d(c \cdot f(x)) = c \cdot df(x).$$

$$(iii) \frac{d}{dx}(f(x) \cdot g(x)) = \frac{d}{dx}f(x) \cdot g(x) + f(x) \cdot \frac{d}{dx}g(x)$$

$$\text{or } d(f(x) \cdot g(x)) = df(x) \cdot g(x) + f(x) \cdot dg(x).$$

$$(iv) \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{\frac{d}{dx}f(x) \cdot g(x) - f(x) \frac{d}{dx}g(x)}{[g(x)]^2}$$

$$\text{or } d \left(\frac{f(x)}{g(x)} \right) = \frac{df(x) \cdot g(x) - f(x) \cdot dg(x)}{[g(x)]^2}.$$

$$(v) \quad z = g(y), \quad y = f(x), \quad \text{i.e., } z = g(f(x)), \quad dz = g'(y)dy,$$

$$dy = f'(x)dx, \quad dz = g'(y) \cdot f'(x)dx.$$

Thus the chain rule of differentiation, namely

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

becomes formally obvious.