

For the following numbers, make the obvious linear approximation, to approximate the given number in decimal form. Then use a calculator or computer to see how close you are using the calculator.

Example: $\sqrt{102}$. The function is $f(x) = \sqrt{x}$. Since $\sqrt{100} = 10$, we think of $102 = 100 + 2$ and 2 is our small increment (Δx or h in texts). $\sqrt{102}$ is close to 10 with a correction term given by the derivative $f'(100)$. We have that the derivative of \sqrt{x} is $f'(x) = (1/2)x^{-1/2}$. Thus $f'(100) = (1/2)(1/10)$. The linear approximation then yields that

$$f(102) \cong f(100) + f'(100)(2) = 10 + [(1/2)(1/(10))](2) = 10 + 1/10 = 10.1$$

A calculator yields $\sqrt{102} = 10.0995049383$. We are off by about $.0005 = 5 \cdot 10^{-4}$, this despite an apparently large δx of 2.

WARNING: there will be a problem much like this on the midterm and likely the final. If this were the problem on the midterm and you gave as your answer only the "exact" (= calculator) answer of 10.0995049383 for your answer you would be getting precisely 0 points for your score on the problem.

M1 . $(1003)^{1/3}$

M2. $\sin(\pi/100)$. (Approximate $\pi/100 = .0314$)

M3. $(2.03)^5$. [use $2^5 = 32$]