Exercises

In Exercises 1–4, find the slope, the y-intercept, and the x-intercept of the line with the given equation.

1. $y = 3x + 12$

2. $y = 4 - x$

3. $4x + 9y = 3$

4. $y - 3 = \frac{1}{2}(x - 6)$

In Exercises 5–8, find the slope of the line.

5. $y = 3x + 2$

6. $y = 3(x - 9) + 2$

7. $3x + 4y = 12$

8. $3x + 4y = -8$

In Exercises 9–20, find the equation of the line with the given description.

9. Slope 3, y-intercept 8

10. Slope −2, y-intercept 3

11. Slope 3, passes through $(7, 9)$

12. Slope −5, passes through $(0, 0)$

13. Horizontal, passes through $(0, -2)$

14. Passes through $(-1, 4)$ and $(2, 7)$

15. Parallel to $y = 3x - 4$, passes through $(1, 1)$

16. Passes through $(1, 4)$ and $(12, -3)$

17. Perpendicular to $3x + 5y = 9$, passes through $(2, 3)$

18. Vertical, passes through $(-4, 9)$

19. Horizontal, passes through $(8, 4)$

20. Slope 3, x-intercept 6

21. Find the equation of the perpendicular bisector of the segment joining $(1, 2)$ and $(5, 4)$ (Figure 12). *Hint:* The midpoint Q of the segment joining (a, b) and (c, d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

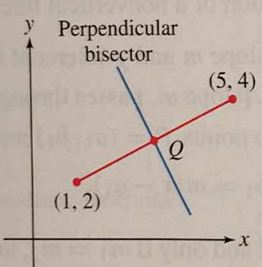

FIGURE 12

22. Intercept-Intercept Form Show that if $a, b \neq 0$, then the line with x-intercept $x = a$ and y-intercept $y = b$ has equation (Figure 13)

$$\frac{x}{a} + \frac{y}{b} = 1$$

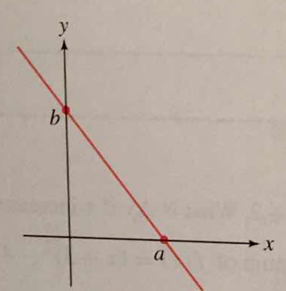

FIGURE 13

23. Find an equ… $y = 3$.

24. Find y such that $(3, y)$ lies on the line of slope $m = 2$ th… $(1, 4)$.

25. Determine whether there exists a constant c such that the $x + cy = 1$:

(a) has slope 4.

(b) passes through (3, 1).

(c) is horizontal.

(d) is vertical.

26. Assume that the number N of concert tickets that can be sold … price of P dollars per ticket is a linear function $N(P)$ for $10 \leq P$ … Determine $N(P)$ (called the demand function) if $N(10) = 50$… $N(40) = 0$. What is the decrease ΔN in the number of tickets so… the price is increased by $\Delta P = 5$ dollars?

27. Suppose that the number of a certain type of computer tha… be sold when its price is P (in dollars) is given by a linear fun… $N(P)$. Determine $N(P)$ if $N(1000) = 10{,}000$ and $N(1500) = 7$… What is the change ΔN in the number of computers sold if the pri… increased by $\Delta P = 100$ dollars?

28. Suppose that the demand for Colin's kidney pies is linear in … price P. Determine the demand function N as a function of P gi… the number of pies sold when the price is P if he can sell 100 pies … the price is \$5.00 and he can sell 40 pies when the price is \$10.00… termine the revenue ($N \times P$) for prices $P = 5, 6, 7, 8, 9, 10$ and … choose a price to maximize the revenue.

29. Materials expand when heated. Consider a metal rod of lengt… at temperature T_0. If the temperature is changed by an amount ΔT, … the rod's length approximately changes by $\Delta L = \alpha L_0 \Delta T$, wher… the thermal expansion coefficient and ΔT is not extreme temper… change. For steel, $\alpha = 1.24 \times 10^{-5}\,°C^{-1}$.

(a) A steel rod has length $L_0 = 40$ cm at $T_0 = $ …°C. Find its le… at $T = 90°C$.

(b) Find its length at $T = 50°C$ if its length at $T_0 = 100°C$ is 6…

(c) Express length L as a function of T if $L_0 = 65$ cm at $T_0 = 1$…

30. Do the points $(0.5, 1)$, $(1, 1.2)$, $(2, 2)$ lie on a line?

31. Find b such that $(2, -1)$, $(3, 2)$, and $(b, 5)$ lie on a line.

32. Find an expression for the velocity v as a linear function of … matches the following data:

t (s)	0	2	4	6
v (m/s)	39.2	58.6	78	97.4

33. The period T of a pendulum is measured for pendulums of s… different lengths L. Based on the following data, does T appear … a linear function of L?

L (cm)	20	30	40	50
T (s)	0.9	1.1	1.27	1.42

34. Show that f is linear of slope m if and only if

$$f(x + h) - f(x) = mh \quad \text{(for all } x \text{ and } h\text{)}$$

That is to say, prove the following two statements:

(a) f is linear of slope m implies that $f(x + h) - f(x)$ … (for all x and h).

(b) $f(x +$ … of slope m.

35. Find th… (a) $f(x) =$ …

In Exercise… imum valu…

36. $y = x$…

38. $y = -$…

40. $y = 2$…

42. $y = 3$…

44. Sketc… minimum…

45. Sketc… point, the…

46. If the… lation wit… and 1), th… alleles) is… heterozyg…

47. For v… root? No r…

48. … the follow… (a) There… roo… (b) … roo…

49. … $(x$ … x…

50. Let a … the arithm… in Exercis…

Further…

55. Show… of fg?

56. Show… and $f(1)$…

57. Show… $[x_1, x_2]$ i… exact dep…

58. Com… mula for…

(b) $f(x+h) - f(x) = mh$ (for all x and h) implies that f is linear of slope m.

35. Find the roots of the quadratic polynomials:

(a) $f(x) = 4x^2 - 3x - 1$ **(b)** $f(x) = x^2 - 2x - 1$

In Exercises 36–43, complete the square and find the minimum or maximum value of the quadratic function.

36. $y = x^2 + 2x + 5$ **37.** $y = x^2 - 6x + 9$

38. $y = -9x^2 + x$ **39.** $y = x^2 + 6x + 2$

40. $y = 2x^2 - 4x - 7$ **41.** $y = -4x^2 + 3x + 8$

42. $y = 3x^2 + 12x - 5$ **43.** $y = 4x - 12x^2$

44. Sketch the graph of $y = x^2 - 6x + 8$ by plotting the roots and the minimum point.

45. Sketch the graph of $y = x^2 + 4x + 6$ by plotting the minimum point, the y-intercept, and one other point.

46. If the alleles A and B of the cystic fibrosis gene occur in a population with frequencies p and $1 - p$ (where p is a fraction between 0 and 1), then the frequency of heterozygous carriers (carriers with both alleles) is $2p(1-p)$. Which value of p gives the largest frequency of heterozygous carriers?

47. For which values of c does $f(x) = x^2 + cx + 1$ have a double root? No real roots?

48. Let f be a quadratic function and c a constant. Which of the following statements is correct? Explain graphically.

(a) There is a unique value of c such that $y = f(x) - c$ has a double root.

(b) There is a unique value of c such that $y = f(x - c)$ has a double root.

49. Prove that $x + \frac{1}{x} \geq 2$ for all $x > 0$. *Hint:* Consider $(x^{1/2} - x^{-1/2})^2$.

50. Let $a, b > 0$. Show that the *geometric mean* $\sqrt{ab}$ is not larger than the *arithmetic mean* $(a+b)/2$. *Hint:* Use a variation of the hint given in Exercise 49.

51. If objects of weights x and w_1 are suspended from th Figure 14(A), the cross-beam is horizontal if $bx = aw_1$. I a and b are known, we may use this equation to determine weight x by selecting w_1 such that the cross-beam is horizc b are not known precisely, we might proceed as follows. x by w_1 on the left as in (A). Then switch places and bala on the right as in (B). The average $\bar{x} = \frac{1}{2}(w_1 + w_2)$ give for x. Show that $\bar{x}$ is greater than or equal to the true wei

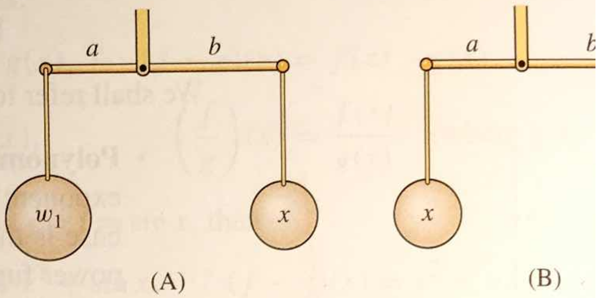

FIGURE 14

52. Find numbers x and y with sum 10 and product 24 quadratic polynomial satisfied by x.

53. Find a pair of numbers whose sum and product are bo

54. Show that the parabola $y = x^2$ consists of all poin $d_1 = d_2$, where d_1 is the distance from P to $(0, \frac{1}{4})$ and tance from P to the line $y = -\frac{1}{4}$ (Figure

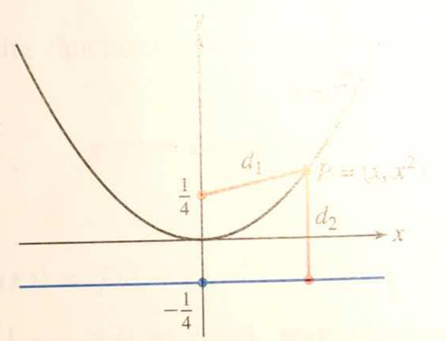

FIGURE 15

Further Insights and Challenges

55. Show that if f and g are linear, then so is $f + g$. Is the same true of fg?

56. Show that if f and g are linear functions such that $f(0) = g(0)$ and $f(1) = g(1)$, then $f = g$.

57. Show that $\Delta y/\Delta x$ for the function $f(x) = x^2$ over the interval $[x_1, x_2]$ is not a constant, but depends on the interval. Determine the exact dependence of $\Delta y/\Delta x$ on x_1 and x_2.

58. Complete the square and use the result to derive the quadratic formula for the roots of $ax^2 + bx + c = 0$.

59. Let $a, c \neq 0$. Show that the roots of

$$ax^2 + bx + c = 0 \quad \text{and} \quad cx^2 + bx + a$$

are reciprocals of each other.

60. Show, by completing the square, that the parabola

$$y = ax^2 + bx + c$$

is congruent to $y = ax^2$ by a vertical and horizontal tra

61. Prove **Viète's Formulas**: The quadratic polynomia as roots is $x^2 + bx + c$, where $b = -\alpha - \beta$ and $c = \alpha\beta$

In Exercises 27–34, calculate the composite functions $f \circ g$ and $g \circ f$, and determine their domains.

27. $f(x) = \sqrt{x}, \quad g(x) = x + 1$

28. $f(x) = \dfrac{1}{x}, \quad g(x) = x^{-4}$

29. $f(x) = 2^x, \quad g(x) = x^2$

30. $f(x) = |x|, \quad g(\theta) = \sin\theta$

31. $f(\theta) = \cos\theta, \quad g(x) = x^3 + x^2$

32. $f(x) = \dfrac{1}{x^2+1}, \quad g(x) = x^{-2}$

33. $f(t) = \dfrac{1}{\sqrt{t}}, \quad g(t) = -t^2$

37.

$$f(x) = \begin{cases} x^2 & \text{whe} \\ -x^2 & \text{whe} \end{cases}$$

38.

$$f(x) = \begin{cases} 2x - 2 & \text{wh} \\ x & \text{wh} \end{cases}$$

39. The population (in millions) of a co (years) is $P(t) = 30 \cdot 2^{0.1t}$. Show that the years. Show more generally that for any p function $g(t) = a2^{kt}$ doubles after $1/k$ y

40. Find all values of c such that $f(x) =$

Further Insights and Challenges

In Exercises 41–47, we define the first difference δf *of a function f by* $\delta f(x) = f(x+1) - f(x)$.

41. Show that if $f(x) = x^2$, then $\delta f(x) = 2x + 1$. Calculate δf for $f(x) = x$ and $f(x) = x^3$.

42. Show that $\delta(10^x) = 9 \cdot 10^x$ and, more generally, that $\delta(b^x) = (b-1)b^x$.

43. Show that for any two functions f and g, $\delta(f + g) = \delta f + \delta g$ and $\delta(cf) = c\delta(f)$, where c is any constant.

44. Suppose we can find a function P such that $\delta P(x) = (x+1)^k$ and $P(0) = 0$. Prove that $P(1) = 1^k$, $P(2) = 1^k + 2^k$, and, more generally, for every whole number n,

$$P(n) = 1^k + 2^k + \cdots + n^k \qquad \boxed{1}$$

45. First show that

$$P(x) = \frac{x(x+1)}{2}$$

satisfies $\delta P = (x + 1)$. Then apply

$$1 + 2 + 3$$

46. Calculate $\delta(x^3)$, $\delta(x^2)$, of degree 3 such that $\delta P = (x+1)^2$ $P(n) = 1^2 + 2^2 + \cdots + n^2$.

47. This exercise combined with Exerci numbers k, there exists a polynomial P tion requires the Binomial Theorem and pendix C).

(a) Show that $\delta(x^{k+1}) = (k+1)\,x^k +$ terms involving smaller powers of x.

(b) Show by induction that there exists with leading coefficient $1/(k+1)$:

$$P(x) = \frac{1}{k+1}x^{k+}$$

such that $\delta P = (x+1)^k$ and $P(0) = 0$.

1.4 Trigonometric Functions

We begin our trigonometric review by recalling the two syste **radians** and **degrees**. They are best described using the relati rotation. As is customary, we often use the lowercase Greek angles and rotations.