loose ends

"infinitesimal"

$\int$

$$f(x+dx) = f(x) + f'(x)dx$$

$$f(g(x+dx)) = f(g(x) + g'(x)dx)$$

$$= f(g(x)) + f'(g(x))g'(x)dx$$

alg of $dx \neq 0$, but $dx^2 = 0$

eg. $f(x) = x^2$.

$dx = .01$

$$f(x+dx) = (x+dx)^2$$

$$= x^2 + 2x\,dx + dx^2.$$

$$= \underline{x^2 + 2x \cdot .01} + (.01)^2$$

thousand

$dx = 10^{-6}$.

$$f(x+dx) = \underline{x^2 + 2x \cdot dx} + 10^{-12}.$$

much smaller than, as far $x = O(1)$

8

$f(x)g(x)$

$f(x+dx)g(x+dx)$

$= (f(x) + f'(x)dx)(g(x) + g'(x)dx)$

$= f(x)g(x) + f'(x)g(x)d$

$+ f(x)g'(x)dx$

$+ \cancel{f'(x)g'(x)dxdx}$

$\leftarrow 0$

argue

"$dxdx << dx$"

so set $= 0$.

or $dx < \frac{1}{n}$ $\forall n$

But $dx \neq 0$.

while $dx^2 = 0$.

$$\frac{f(x+dx)}{g(x+dx)} = \frac{f(x)+f'(x)dx}{g(x)+g'(x)dx}$$

Now $(a+b)(a-b) = a^2 - b^2$.

$$\frac{(f(x)+f'(x)dx)(g(x)-g'(x)dx)}{(g(x)+g'(x)dx)(g(x)-g'(x)dx)}$$

$$= \frac{f(x)g(x) + [f'(x)g(x) - f(x)g'(x)]dx}{g(x)^2 - g'(x)^2 dx^2}.$$

Set $dx^2 = 0$

$$\Rightarrow \cancel{f(x)g}$$

$$= \frac{f(x)}{g(x)} + \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}dx.$$

Ex

$$f(x) = x^n$$

$$f(x+dx) = (x+dx)^n$$

$$= (x+dx)(x+dx)\cdots(x+dx)$$

$$= x^n + nx^{n-1}dx + \underbrace{dx^2\text{'s}}_{0}$$

$$= x^n + nx^{n-1}dx$$

so $f'(x) = nx^{n-1}$,